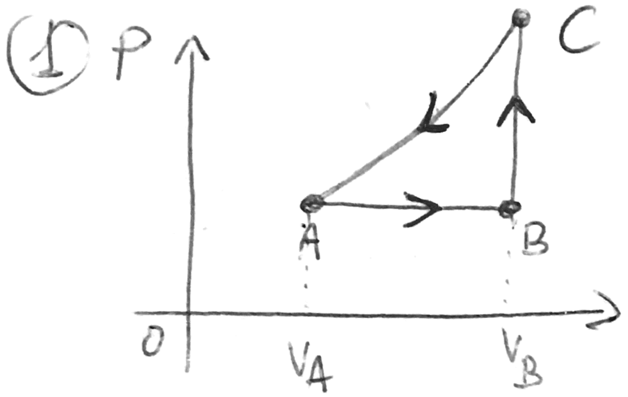
SVOLGIMENTO PROVA DEL 16/10/2017 (A)

(1)



$C \to A \quad P(V) = aV^3$

$P_A = aV_A^3 \qquad P_C = aV_C^3 = aV_B^3$

$T_A = \frac{aV_A^4}{nR} \qquad T_C = \frac{aV_C^4}{nR} = \frac{aV_B^4}{nR}$

Essendo S una funzione di stato:

$$\Delta S_{CA} = -(\Delta S_{AB} + \Delta S_{BC}) = -\int_A^B \frac{\delta Q}{T} - \int_B^C \frac{\delta Q}{T} =$$

$$= -\int_{T_A}^{T_B} \frac{n c_p \, dT}{T} - \int_{T_B}^{T_C} \frac{n c_v \, dT}{T} = -n c_p \ln\frac{T_B}{T_A} - n c_v \ln\frac{T_C}{T_B}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{P_A V_B}{nR} = \frac{a V_A^3 V_B}{nR};$$

$$\Delta S_{CA} = n c_p \ln\left(\cancel{\frac{aV_A^4}{nR}} \cancel{\frac{nR}{aV_A^3 V_B}}\right) + n c_v \ln\left(\cancel{\frac{aV_A^3 V_B}{nR}} \cancel{\frac{nR}{aV_B^4}}\right)$$

$$= n c_p \ln\left(\frac{V_A}{V_B}\right) + n c_v \ln\left(\frac{V_A}{V_B}\right)^3$$

$$= n(c_p + 3c_v) \ln\left(\frac{V_A}{V_B}\right) = nR\left(\frac{7}{2} + 3\cdot\frac{5}{2}\right) \ln\left(\frac{V_A}{V_B}\right) =$$

$$= 8.31 \cdot 11 \cdot \ln(2/5) \text{ J/K} = -83{,}76 \text{ J/K};$$

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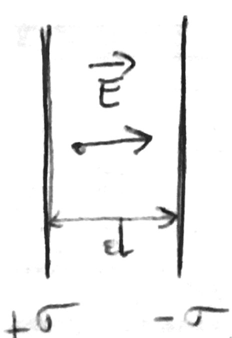
(2)



$|\vec{E}| = \frac{\sigma}{\varepsilon_0} \qquad \Delta V = |\vec{E}| d \to |\vec{E}| = \frac{\Delta V}{d}$

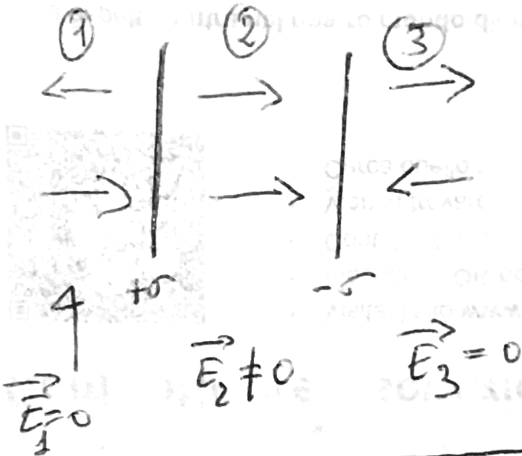
$|\vec{E}| = \frac{120 \text{ V}}{2\cdot 10^{-2} \text{ m}} = 60\cdot 10^2 \text{ N/C} = 6\cdot 10^3 \text{ N/C};$

$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = |q| \Delta V = 120 \text{ eV};$

$|\vec{F}_e| = |q_e||\vec{E}| = |q_e| \Delta V / d; \quad |\vec{F}_g| = m_e g$

$$|\vec{F}_{el}|/|\vec{F}_g| = \frac{|q_e|\Delta V}{m_e g d} = 1{,}07\cdot10^{14};$$

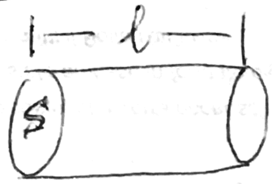
B



$$\sigma = \frac{Q}{S} \qquad |\vec{E}| = \frac{\sigma}{\varepsilon_0}$$

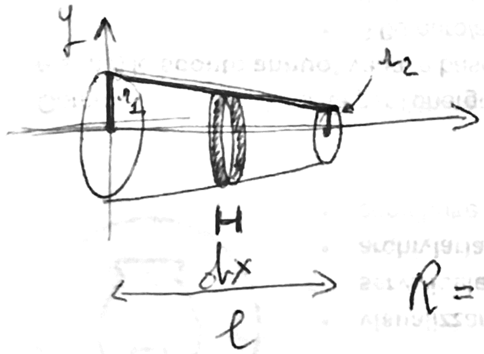
$$S = \frac{Q}{\varepsilon_0|\vec{E}|} = \frac{Q\,d}{\varepsilon_0\Delta V} = 1{,}88\cdot10^{7}\,m^2;$$

3



$l = 10\,cm$; $S = 2\,cm^2$ $\quad R = 1\,k\Omega$

$$R_0 = \rho\frac{l}{S} \longrightarrow \rho = \frac{S R}{l} = 2\,\Omega m$$



$$dR = \rho\frac{dx}{S} = \frac{\rho}{\pi}\frac{dx}{y(x)^2}$$

$$y(x) = \frac{r_2-r_1}{l}x + r_1$$

$$R = \frac{\rho}{\pi}\int_0^l \frac{dx}{\left(\frac{r_2-r_1}{l}x+r_1\right)^2} = \frac{\rho}{\pi}\,\frac{1}{\frac{r_2-r_1}{l}x+r_1}\,\frac{l}{r_1-r_2}\Bigg|_0^l$$

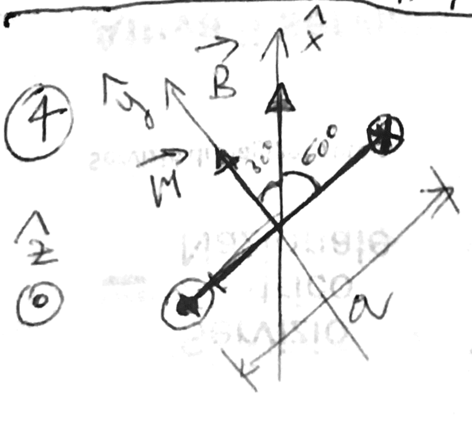
$$= \frac{\rho}{\pi}\left\{\frac{l}{(r_1-r_2)\left[r_2-\cancel{r_1}+\cancel{r_1}\right]} - \frac{l}{(r_1-r_2)\,r_1}\right\} =$$

$$= \frac{\rho}{\pi}\frac{l}{\cancel{r_1-r_2}}\left\{\frac{\cancel{r_1-r_2}}{r_1r_2}\right\} = \frac{\rho}{\pi}\frac{l}{r_1r_2} = \frac{S R_0}{\cancel{l}}\frac{\cancel{l}}{\pi}\frac{1}{r_1r_2}$$

$$\Longrightarrow R = R_0\frac{S}{\pi r_1 r_2} = 10^3\,\Omega\,\frac{\cancel{2}\cdot10^{-4}\,m}{3{,}14\,\cancel{2}\cdot10^{-2}\cdot10^{-2}\,m} = 318{,}31\,\Omega$$

Valida nell'ipotesi di piccole

4



$30° \to \pi/6 < 1$

oscillazioni

$$\vec{m} = Iab\,\hat{y} \qquad \vec{B} = B_0\hat{x}$$

$$\vec{M} = \vec{m}\times\vec{B} = IB_0ab\,\hat{y}\times\hat{x} = IB_0ab\sin\theta\,\hat{z}$$

$$\vec{M} = \mathbb{I}\vec{\alpha} \longrightarrow \boxed{IB_0ab\sin\theta = -J\ddot{\theta}}$$

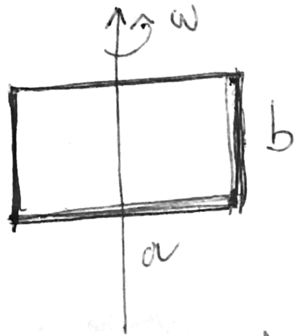
(c)

$$\ddot{\theta} + \left(\frac{I B_0 a b}{J}\right)\theta = 0$$

$\frac{I B_0 a b}{J} \longrightarrow \omega_0^2$ pulsazione

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{J/I B_0 a b}\ ;$$

Calcoliamo il momento di inerzia della spira. Sia μ la sua massa che possiamo supporre distribuita uniformemente lungo la spira.



Densità di massa $\lambda = \frac{\mu}{2(a+b)}$

$$J = 2(J_a + J_b) = 2\left(\frac{1}{12}\lambda a^3 + \lambda b\left(\frac{a}{2}\right)^2\right) =$$

$$= \frac{\lambda a^2}{2}\left(\frac{a}{3} + b\right) = \frac{\mu}{4(a+b)} a^2 \frac{a+3b}{3} = \frac{1}{12}\left(\frac{a+3b}{a+b}\right)\mu a^2$$

$$T' = 2\pi\sqrt{\frac{(a+3b)\mu a^2}{12(a+b)\, I B_0 \cancel{a} b}} = \pi\sqrt{\frac{(a+3b)\mu a}{3(a+b) I B_0 b}}$$

$$= 1{,}09\ s^{-1};$$

$$\frac{1}{2} J \omega^2 = \cancel{} = \int_{\pi/6}^{0} |\vec{m}| B_0 \sin\theta \, d\theta =$$

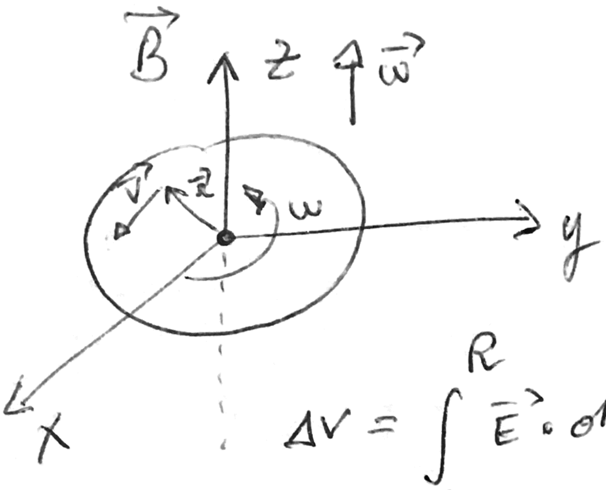
$$= -|\vec{m}| B_0 (-\cos\theta)\Big|_{\pi/6}^{0} = -|\vec{m}| B_0 (\cos \pi/6 - 1) = |\vec{m}| B_0\left(-\frac{\sqrt{3}}{2} + 1\right)$$

$$\omega^2 = \frac{2|\vec{m}| B_0 (-\sqrt{3}/2 + 1)}{J};$$

$$\omega = \sqrt{\frac{24 I a b B_0 (-\sqrt{3}/2 + 1)(a+b)}{(a+3b)\mu a^2}} = 2{,}96\ \text{rad/s}$$

bisogna applicare una coppia in modulo pari a $|\vec{m}||\vec{B}| \sin \pi/6$ e di verso contrario a $\vec{m} \times \vec{B}$.

5 — D



$$\vec{E}_{IN} = \vec{v} \times \vec{B} = (\vec{\omega} \times \vec{r}) \times \vec{B} = + \omega \vec{r} B_z$$

$$\Delta V = \int_0^R \vec{E} \cdot d\vec{r} = \int_0^R \omega B_0 r \, dr = \frac{1}{2} \omega B_0 R^2$$

$$R^2 = \frac{2 \Delta V}{\omega B_0} \Rightarrow R = \sqrt{\frac{2 \Delta V}{\omega B_0}} = 0{,}47 \text{ m}.$$