Maps for the Electron Cloud

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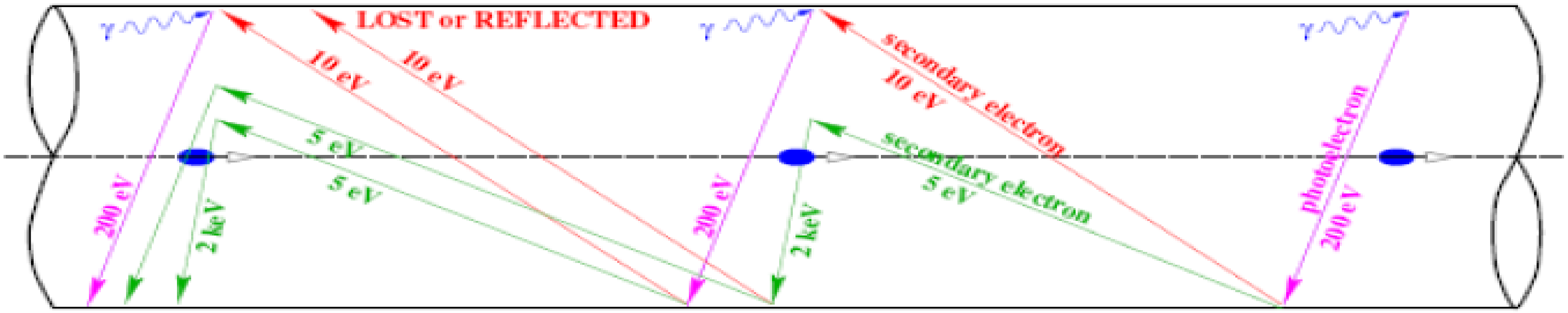
Plan of Talk

1. Electron cloud effect
2. Buildup of electron cloud and the map formalism
3. The linear coefficient
4. Model of space charge in magnetic field
5. The quadratic coefficient
6. Conclusions and Outlook.

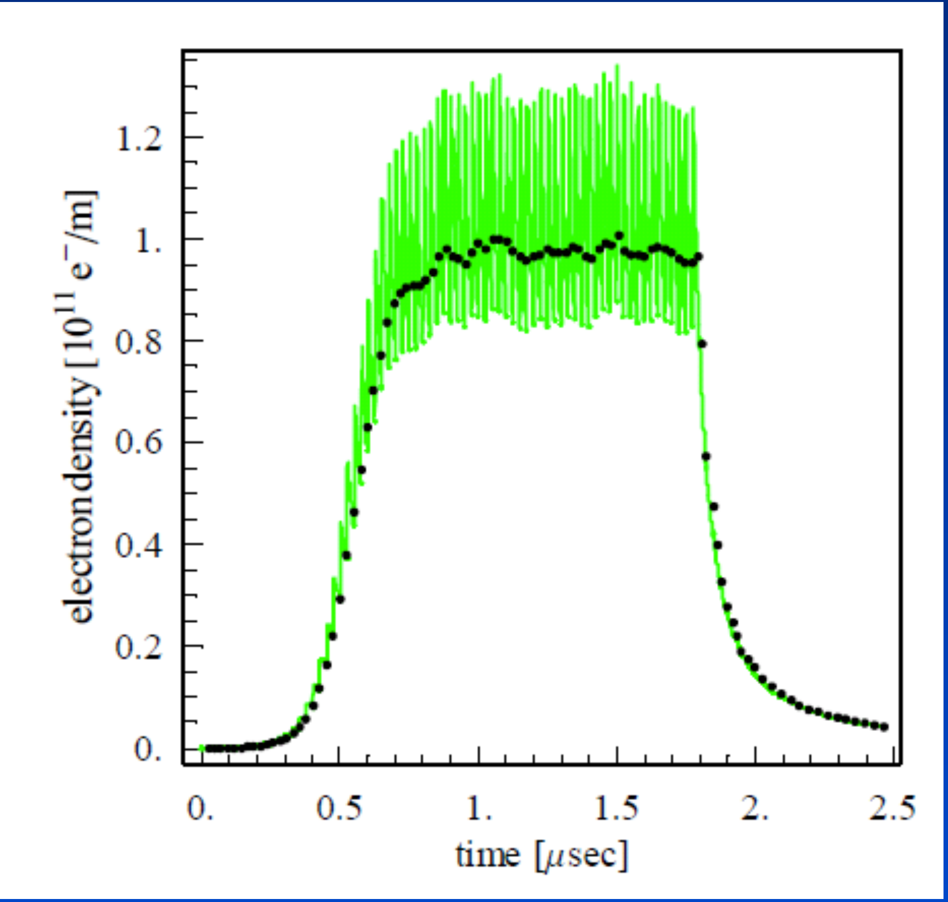
Electron Cloud Effect



The electron cloud develops quickly as photons striking the vacuum chamber wall knock out electrons that are then accelerated by the beam, gain energy, and strike the chamber again, producing more electrons.

The interaction between the electron cloud and a beam leads to the electron cloud effects such as single- and multi-bunch instability, tune shift, increase of pressure and so on.

Eclectron Cloud Buildup



LHC dipole parameters

parameter	unit	value
beam particle energy	GeV	7000
bunch spacing	ns	25
bunch length	m	0.075
number of bunches N_b	–	72
number of particles per bunch N	10^{11}	0.8 to 1.6
bending field B	T	8.4
length of bending magnet	m	14.2
vacuum screen half height	m	0.018
vacuum screen half width	m	0.022
circumference	m	27000
primary photo-emission yield	-	$7.98 \cdot 10^{-4}$
maximum SEY δ_{max}	-	1.3 to 1.7
energy for max. SEY E_{max}	eV	237.125
energy width for secondary e^{-}	eV	1.8

— ECLOUD (CERN) output

• bunch to bunch average

Maps Formalism

For a given beam pipe characteristics the evolution of the electron density is only driven by the bunch passing by, and the existing electron density before the bunch passage:

$$\rho_{m+1} = F(\rho_m)$$

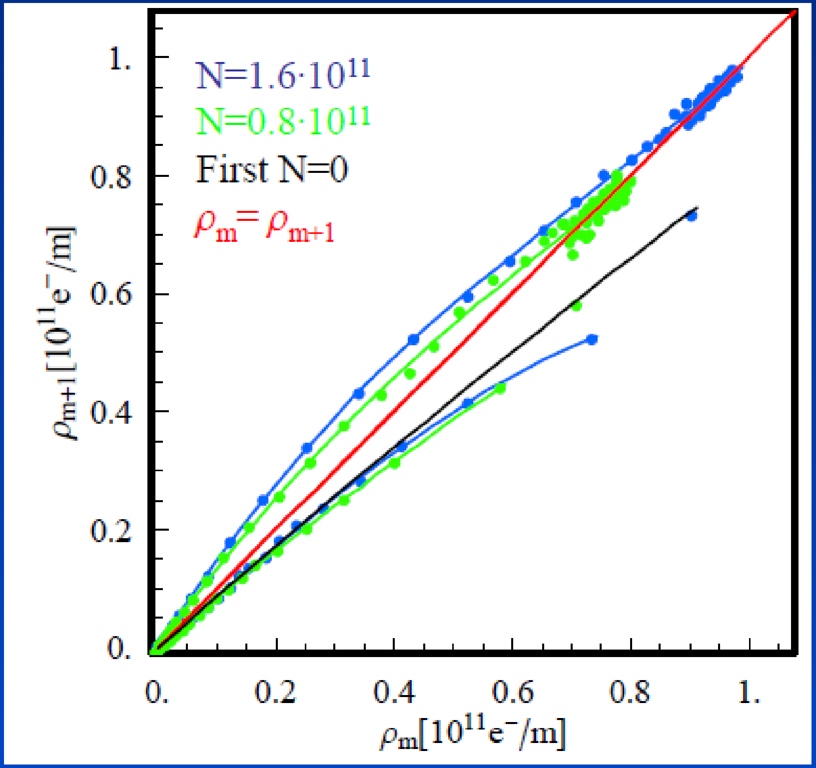
Simplify the e-cloud problem into a small number of mathematical parameters.

The bunch-to-bunch evolution of the e-cloud density is represented by a cubic map:

$$\rho_{m+1} = a\,\rho_m + b\,\rho_m^2 + c\,\rho_m^3$$

where ρ_m is the bunch to bunch average of the electron line density

Building the Cubic Map



Lines corresponds to cubic fit of the form:

$$\rho_{m+1} = a\,\rho_m + b\,\rho_m^{2} + c\,\rho_m^{3}$$

Three sets of coefficients are needed to describe the ecloud density evolution

Analytical Determination of Linear Coefficient

The total number of low energy electrons at the arrival of (m+1) is given by

$$N_{m+1}(\mathcal{E}_0) = N_m \delta_s(\mathcal{E}_g) \cdot \sum_{p=1}^{S} \delta_r^{p-1}(\mathcal{E}_g)\left[\delta_t(\mathcal{E}_0)+\delta_r(\mathcal{E}_0)\right]^{k_p}$$

The total number of (fast and slow) electrons at the of (m+1) is given by

$$N_{m+1} = N_m \left[\delta_r{}^S(\mathcal{E}_g)+\delta_t(\mathcal{E}_g) \cdot \sum_{p=1}^{S} \delta_r{}^{p-1}(\mathcal{E}_0)\delta_{tot}^{k_p}(\mathcal{E}_0)\right]$$

The linear coefficient is given by

$$\alpha = \frac{N_{m+1}}{N_m} = \delta_r^S(\mathcal{E}_g) + \delta_t(\mathcal{E}_g)\delta_{tot}^{\eta}(\mathcal{E}_0)\frac{\delta_{tot}^{\eta S}(\mathcal{E}_0)-\delta_r^S(\mathcal{E}_0)}{\delta_{tot}^{\eta}(\mathcal{E}_0)-\delta_r(\mathcal{E}_0)}$$

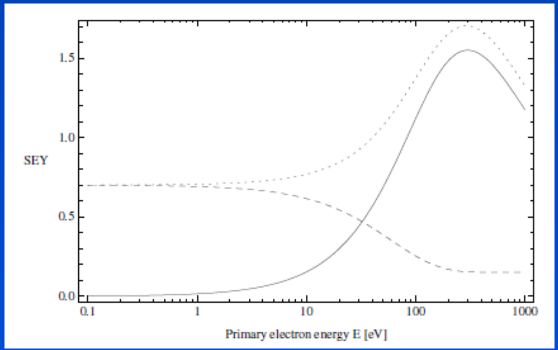
$$\eta = t_f(\mathcal{E}_g)/t_f(\mathcal{E}_0) = (\mathcal{E}_0/\mathcal{E}_g)^{1/2}$$

$$\delta_{tot} = \delta_r + \delta_s$$

$$\delta_{ts}(E) = \delta^*_{max}\frac{E}{E_{max}}\frac{s}{s-1+\left[\frac{E}{E_{max}}\right]^s}$$

$$\delta_{ref}(E) = \delta_{inf} + (\delta_0 - \delta_{inf})\, e^{-\frac{E}{E_r}}$$

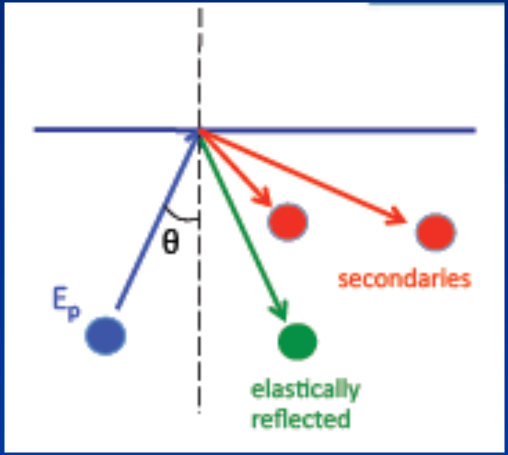
$$\delta_{tot}(E) = \delta_{ts}(E) + \delta_{ref}(E)$$



Parameters	Quantities	Unit	Value
Maximum SEY	δ_{max}	/	1.6
SEY for $E \to 0$	δ_0	/	0.7
SEY for $E \to \infty$	δ_∞	/	0.15
Energy for max δ	E_{max}	eV	300
	E_r	eV	60
	s	/	1.5

… but in presence of magnetic field

The Sey also depends on the angle at which the electrons strike the chamber wall. For non-normal incidence, d_{tot} is multiplied by $\exp(\alpha_p(1-\cos\theta))$, and also the E_{max} is multiplied by $(1+\beta_p(1-\cos\theta))$



$$a = a(E,\mathcal{E}_0,\theta) = \delta_{ref}(E)^k + \delta_{ts}(E)\,\delta_{tot}(\mathcal{E}_0,\theta)^\xi\,\frac{\delta_{tot}(\mathcal{E}_0,\theta)^{k\,\xi} - \delta_{ref}(E)^k}{\delta_{tot}(\mathcal{E}_0,\theta)^\xi - \delta_{ref}(E)}$$

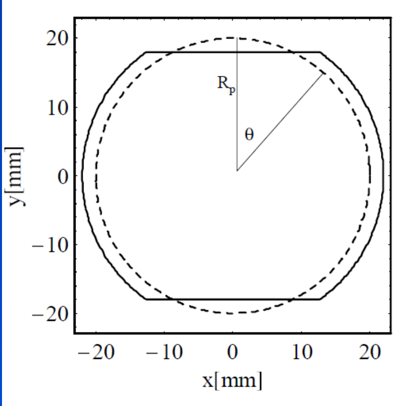
$$k = \frac{s_b/c}{t_f} - 1/2$$

$$\xi = \sqrt{\frac{\mathcal{E}_0}{E_g}}$$

Magnetic Dipole Field : By = 8.4 T

Assumption: the transverse elicoidal motion of the electrons is approximated by a vertical motion (the radius of the particle trajectories is very small compared to the beam pipe radius R_p).

Beam Pipe Geometry

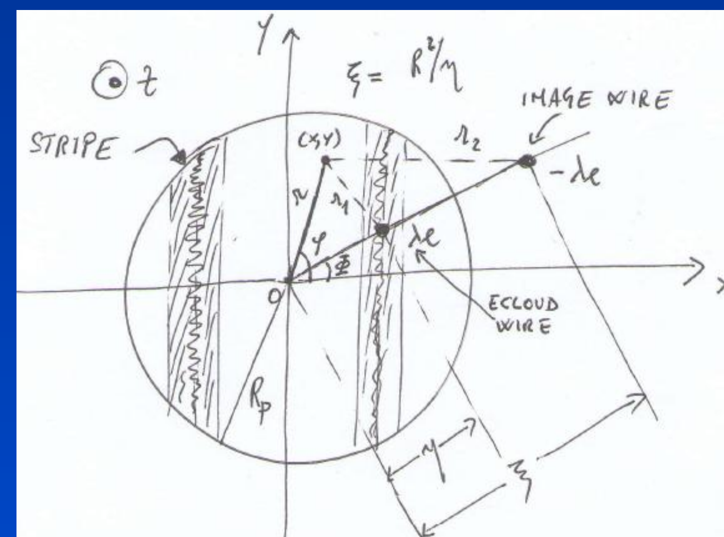


Model of the Space Distribution of Electron Cloud

In presence of magnetic field the space distribution of ecloud can be modeled by

$$\rho_e = \frac{-e\,N_e}{\sigma_e}\,g(x,y) = \frac{-e\,N_e}{\sigma_e}\,\frac{X\left[e^{-\frac{(x-L)^2}{2\Delta L^2}} + e^{-\frac{(x+L)^2}{2\Delta L^2}}\right] + X_c\,e^{-\frac{x^2}{2\Delta L_c{}^2}} + u}{4\int_0^{R_p} dl\sqrt{R_p{}^2 - l^2}\left[X(e^{-\frac{(l-L)^2}{2\Delta L^2}} + e^{-\frac{(l+L)^2}{2\Delta L^2}}) + X_c\,e^{-\frac{l^2}{2\Delta L_c{}^2}} + u\right]}$$

The electrostatic potential generated by an uniform charged wire distribution, satisfying the boundary condition, is



$$v(r,\phi) = \frac{\lambda_e}{2\pi\epsilon_0}\left[\ln\sqrt{\frac{r^2 + \frac{R_p{}^4}{\eta^2} - 2r\frac{R_p{}^2}{\eta}\cos(\Phi-\phi)}{r^2+\eta^2-2\eta r\cos(\Phi-\phi)}} - \ln\frac{R_p}{\eta}\right] = \frac{\lambda_e}{2\pi\epsilon_0}\,f(r,\phi,\eta,\Phi)$$

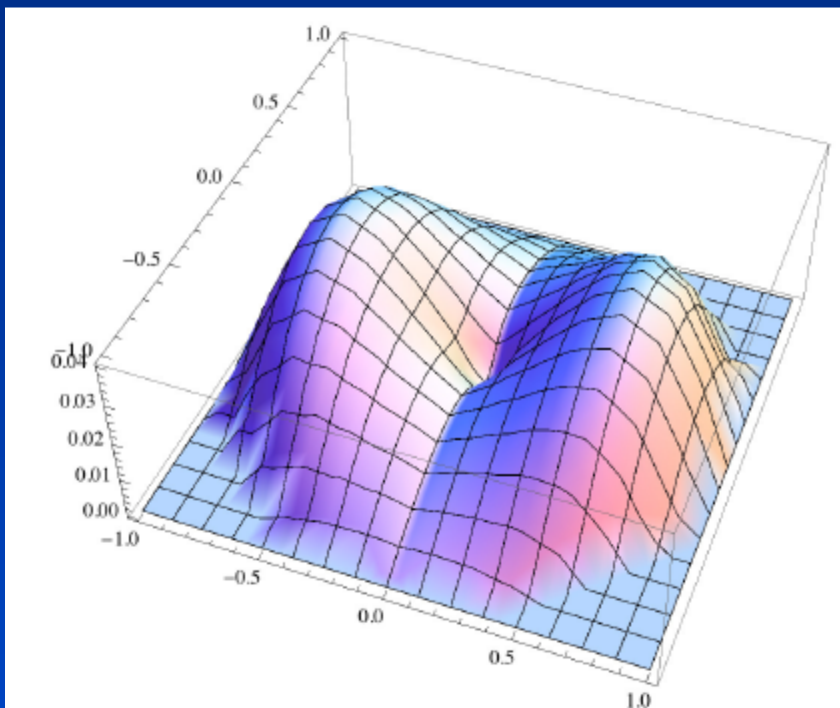
The total potential in the chamber is

$$V(r,\phi) = -\frac{e\,N_e}{2\,\pi\,\epsilon_0\,\sigma_e}\int_{S'} dS'\,g(x',y')\,f(r,\phi,\eta',\Phi') + \frac{e\,N_b}{2\,\pi\,\epsilon_0\,\sigma_z}\left[\left(\frac{1}{2} - \frac{r^2}{2\,\sigma_r{}^2} + \ln\frac{R_p}{\sigma_r}\right)\Theta(\sigma_r - r) + \ln\frac{R_p}{r}\,\Theta(r-\sigma_r)\right]$$

Energy Barrier in Presence of Magnetic Field

The energy barrier

$$\mathcal{E}(r,\phi) = m_e c^2\left\{\frac{2\, r_e\, N_e}{\sigma_e}\, h(r,\phi) - \frac{2\, r_e\, \bar{N}_b}{\sigma_z}\left[\left(\frac{1}{2} - \frac{r^2}{2\,{\sigma_r}^2} + \ln\frac{R_p}{\sigma_r}\right)\Theta(\sigma_r - r) + \ln\frac{R_p}{r}\,\Theta(r-\sigma_r)\right]\right\}$$



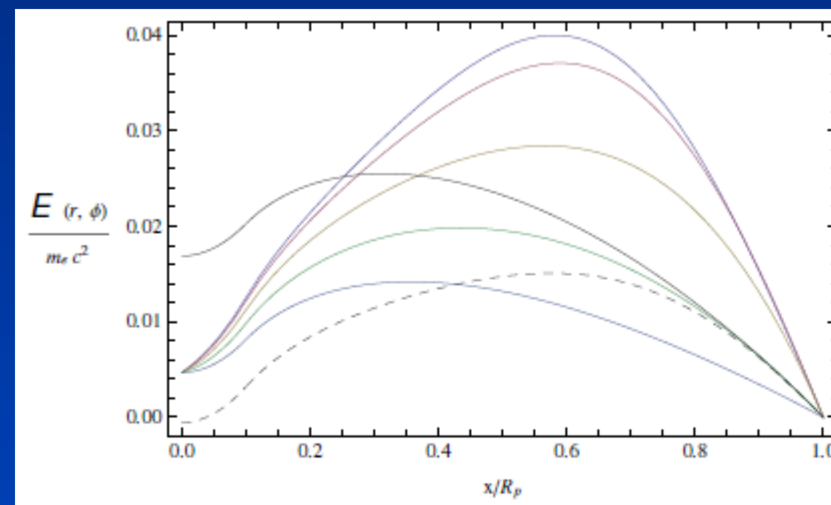


FIG. 2: Plot of energy barrier for fixed values of angle $\phi = 0,\ \pi/8,\ \pi/4,\ 3\pi/8,\ \pi/2$ (colored lines). In the absence of magnetic field we considered the uniform (black line) and the gausslike distribution of space charge (dashed line). In the case of density (4) the values of parameters are $X = 2$, $L = 0.7 * R_p$, $\Delta L = 0.2 * R_p$, $X_c = 0.5 * R_p$, $\Delta L_c = 0.1 * R_p$, $u = 0.01$. For the radially gaussian distribution, $\propto e^{-\frac{(r-L)^2}{2\,\Delta L^2}}$, one set $L = 0.8 * R_p$, $\Delta L = 0.2 * R_p$. For all cases $N_e = 10 * N_b = 10^{11}$, $\sigma_e = \sigma_z$.

Energy Barrier in Absence of Magnetic Field

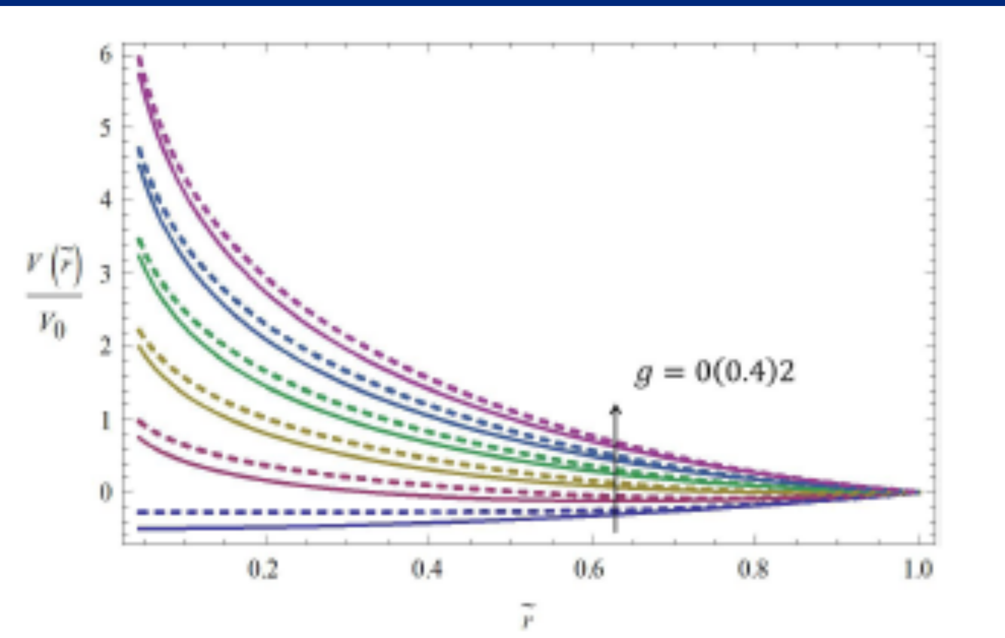
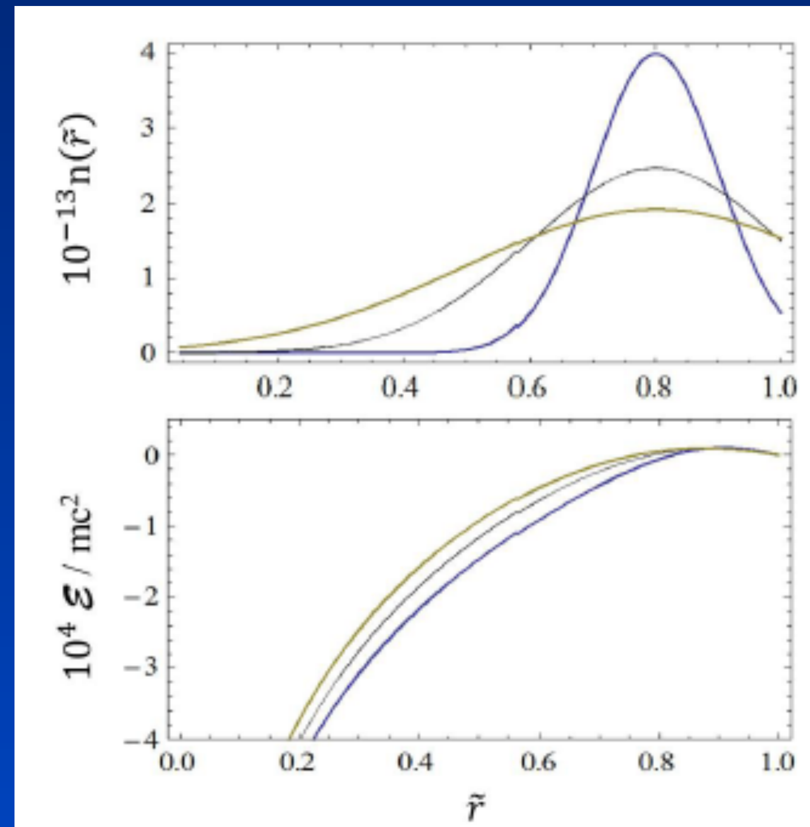


Figure 3: *The potentials in Eq. (9) (dashed lines), and (12) (solid lines) as functions of $\tilde{r}$ for various values of g and $\tilde{a}$=0.04, $\tilde{r}_0$=0.8, $\tilde{\sigma}$=0.2.*



Parameter	Quantity	Unit	Value
Beam pipe radius	b	m	.045
Beam size	a	m	.002
Bunch spacing	s_b	m	1.2
Bunch length	h	m	.013
Particles per bunch	N_b	10^{10}	$4 \div 9$

Figure 4: *The electron density $n(\tilde{r})$ (solid line) and the energy barrier $\mathcal{E}(\tilde{r})$ (dashed line) for $\tilde{\sigma}$ =* 0.1(0.1)0.3, $\tilde{r}_0 = 0.8$, $N_b = 6 \cdot 10^10$.

$n(\tilde{r}) = -\rho(\tilde{r})/e$

Energy gained by the electrons

The electrons are accelerated by the bunch. The energy gain of the electron in the position (x,y) is given by (Kick approximation)

$$\Delta E_{kick}(x,y) = 2 m_e c^2 r_e^2 N_b^2 \frac{y^2}{(x^2+y^2)^2}\Theta(r - R_C)$$

where the critical radius is

$$R_C = 2\sqrt{\sqrt{\frac{2}{\pi}} N_b r_e \sigma_z}$$

The average energy gain

$$E_g = \int_{S'} dS' \, g(x',y') \, \Delta E_{kick}(x',y')$$

The average time of flight

$$t_f = \frac{2 R_p}{\sqrt{\frac{2 E_g}{m_e}}} \int_{S'} dS' \, g(x',y') \sin\Phi'$$

Parameters	Quantities	Unit	Value
Beam pipe radius	R_p	m	0.02
Beam size	σ_r	m	0.002
Bunch spacing	s_b	m	7.48
Bunch length	σ_z	m	0.075
Particles per bunch	N_b	10^{10}	$8 \div 12$
Magnetic filed	B	T	8, 39

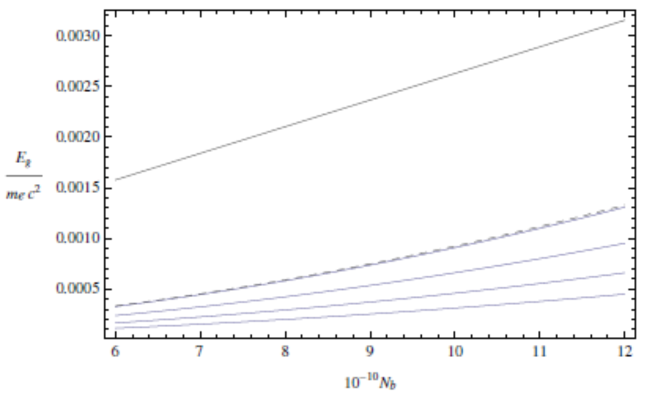


FIG. 3: The energy gain (5) is plotted for the values $\Delta L = 0.2 * R_p$, $X_c = u = 0$, $L = (0.3 - 0.6) * R_p$ with step $0.1 * R_p$ (colored lines). Furthrtmore the solid line if $R_C >> R_p$ while the dashed line if the kick zone is all space outside the bunch.

Saturation condition

The saturation condition for the map gives

$$b = b(E, \mathcal{E}_0, \theta) = \frac{1 - a(E, \mathcal{E}_0, \theta)}{n_e^{sat}}$$

The density of saturation

$$n_e(r, \phi, \mathcal{E}(r,\phi)) = \frac{\frac{\mathcal{E}(r,\phi)}{2\, r_e\, m_e c^2} + \frac{N_b}{\sigma_z}\left[\left(\frac{1}{2} - \frac{r^2}{2\,\sigma_r{}^2} + \ln\frac{R_p}{\sigma_r}\right)\Theta(\sigma_r - r) + \ln\frac{R_p}{r}\,\Theta(r - \sigma_r)\right]}{\pi\, R_p{}^2\, h(r,\phi)}$$

is obtained by imposing the condition in the point

$$\bar{r} = \bar{r}(x) = \sqrt{x^2 + \left(s_b\sqrt{\frac{2\mathcal{E}_0}{m_e c^2}} - R_p\right)^2\left(1 - \frac{x^2}{R_p{}^2}\right)}$$

$$\bar{\phi} = \bar{\phi}(x) = \arctan\frac{\left(s_b\sqrt{\frac{2\mathcal{E}_0}{m_e c^2}} - R_p\right)\sqrt{1 - \frac{x^2}{R_p{}^2}}}{x}$$

The average density is given by

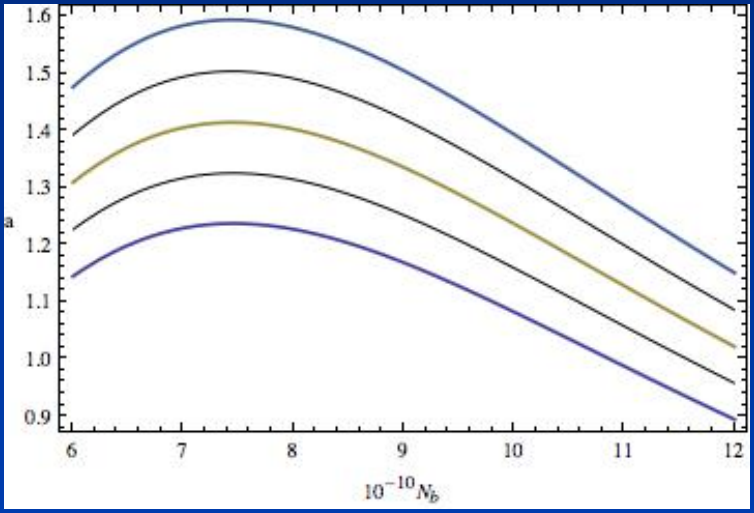
$$n_e^{sat} = \int_{S'} dS'\, g(x', y')\, n_e\left(\bar{r}(x'),\, \bar{\phi}(x'),\, \mathcal{E}_0\left[1 - \frac{x'^2}{R_p{}^2}\right]\right) \sim n_e\left(\bar{r}(L),\, \bar{\phi}(L),\, \mathcal{E}_0\left[1 - \frac{L^2}{R_p{}^2}\right]\right)$$

The coefficients of map are given by

$$a = a\left(E_g, \mathcal{E}_0\left[1 - \frac{L^2}{R_p{}^2}\right], \frac{\pi}{2} - \arccos\frac{L}{R_p}\right)$$

$$b = \frac{1 - a\left(E_g, \mathcal{E}_0\left[1 - \frac{L^2}{R_p{}^2}\right], \frac{\pi}{2} - \arccos\frac{L}{R_p}\right)}{n_e\left(\bar{r}(L),\, \bar{\phi}(L),\, \mathcal{E}_0\left[1 - \frac{L^2}{R_p{}^2}\right]\right)}$$

Coefficients of Map



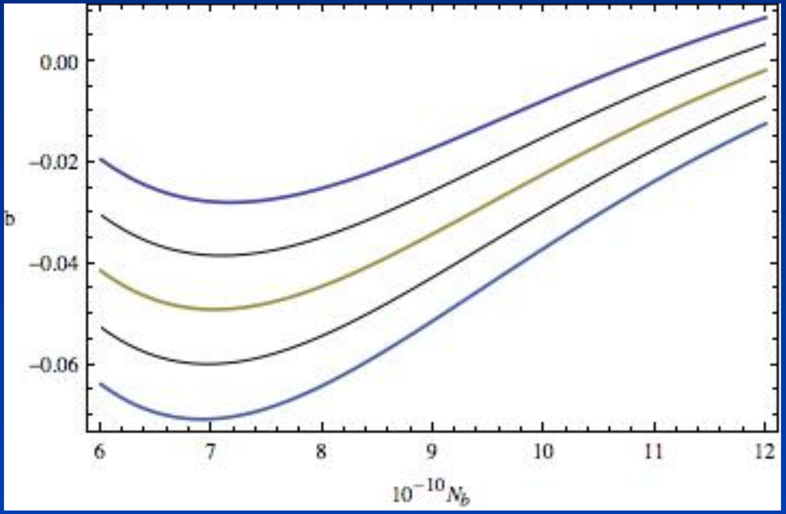


FIG. 5: Comparison between the theoretical behavior and the simulation for the coefficient a (first line of (15)) by setting $\delta_{max} = 1.5$ (blue line and triangles), 1.6 (black line and squares), 1.7 (yellow line and circles). The values of density parameters are $X = 1$, $L = 0.4 * R_p$, $\Delta L = 0.15 * R_p$, $X_c = 0.5$, $\Delta L_c = 0.05 * R_p$, $u = 0$. The values of SEY parameters are shown in Table II.

Conclusions and Outlook

1. The electron-cloud buildup can be described by a quadratic map.

2. An approximate formula has been derived for the quadratic coefficient in presence of magnetic field.

3. The analytical result could be useful to determine safe regions in parameters space where to minimize the electron clouds.