Study of Fourth Order Gravity at low energy by analyzing the gravitational lensing and the galactic rotation curves

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Abstract

For a general class of analytic functions , where is the Ricci scalar, is the Rissi tensor and is the Rienman tensor, the gravitational lensing and the galactic rotation curves are shown. The analysis is performed in the socalled Newtonian limit of theory. From the properties of the Gauss-Bonnet invariant it is enough to consider only one curvature invariant between the Ricci tensor and the Riemann tensor. The metrics are time independent and spherically symmetric. Considering first the case of a pointlike lens, and after the one of a generic matter distribution of the lens, the deflection angle and the angular position of images are studied. Though the additional terms in the gravitational potential modifies dynamics with respect to General Relativity, the geodesic trajectory of the photon is unaffected by the modification if we consider only function. In the case of galactic rotations the spatial behavior of curves has been evalueted adding also the dark matter component. A systematic analysis of rotation curves induced by all galactic substructures of ordinary matter is shown. The outcomes are compared with respect to the classical outcomes of General Relativity. The theoretical spatial behaviors of rotation curve are compared with the experimental data for the Milky Way and the galaxy NGC 3198. Although the Fourth Order Gravity gives more rotational contributions, in the limit of large distances the Keplerian behavior is ever present, and it is missing only if the dark matter component is added. After the analysis of lensing gravitational and galactic rotation curves it remains a hard challenge to interpret the dark matter effects as a single geometric phenomenon.

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1. Fourth Order Gravity: Lagrangian, field equations, solutions
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Fourth Order Gravity (FOG)

The action principle $$\mathcal{A}^f = \int d^4x\sqrt{-g}\Big[f(X,Y,Z) + \mathcal{X}\mathcal{L}_m\Big]$$

where $X = R$ (Ricci scalar), $Y = R_{\alpha\beta}R^{\alpha\beta}$ (Ricci tensor square) and $Z = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ (Riemann square)

By using the metric approach the field equations are

$$\begin{aligned} H_{\mu\nu} = \; & f_X R_{\mu\nu} - \frac{f}{2}g_{\mu\nu} - f_{X;\mu\nu} + g_{\mu\nu}\Box f_X + 2f_Y R_\mu{}^\alpha R_{\alpha\nu} - 2[f_Y R^\alpha{}_{(\mu}]_{;\nu)\alpha} + \Box[f_Y R_{\mu\nu}] + [f_Y R_{\alpha\beta}]^{;\alpha\beta} g_{\mu\nu} \\ & + 2f_Z R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma} - 4[f_Z R_\mu{}^{\alpha\beta}{}_\nu]_{;\alpha\beta} = \mathcal{X}T_{\mu\nu} \end{aligned}$$

where $f_X = \frac{df}{dX}$, $f_Y = \frac{df}{dY}$, $f_Z = \frac{df}{dZ}$ $\qquad \Box = \frac{\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta)}{\sqrt{-g}}$

$$A^{\alpha\beta\ldots\delta}{}_{;\mu} = A^{\alpha\beta\ldots\delta}{}_{,\mu} + \Gamma^\alpha_{\sigma\mu}A^{\sigma\beta\ldots\delta} + \Gamma^\beta_{\sigma\mu}A^{\alpha\sigma\ldots\delta} + \cdots + \Gamma^\delta_{\sigma\mu}A^{\alpha\beta\ldots\sigma}$$

The energy momentun tensor of matter is $T_{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$

The trace equation is

$$H = g^{\alpha\beta}H_{\alpha\beta} = f_X R + 2f_Y R_{\alpha\beta}R^{\alpha\beta} + 2f_Z R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 2f + \Box[3f_X + f_Y R] + 2[(f_Y + 2f_Z)R^{\alpha\beta}]_{;\alpha\beta} = \mathcal{X}T$$

Newtonian limit of FOG – Tools of calculus

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The Lagrangian of the massive particle embedded in the gravitational field is proportional to the “relativistic distance”.

$$L = \left(g_{\rho\sigma}\frac{dx^\rho}{dt}\frac{dx^\sigma}{dt}\right)^{1/2} = \left(g_{tt} + 2g_{tm}v^m + g_{mn}v^m v^n\right)^{1/2}$$

And the geodesic equation is obtained by applying the variational principle

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\sigma\tau}\frac{dx^\sigma}{ds}\frac{dx^\tau}{ds} = 0$$

The approximation level of field equations is driven by an expansion of the powers of $(1/c)^{2n}$ where c is the light speed and n is integer.

At lowest level (n = 1) we have the Newtonian Mechanics

$$\frac{d^2x^i}{dt^2} \simeq -\Gamma^i_{tt} \simeq -\frac{1}{2}\frac{\partial g_{tt}}{\partial x^i}$$

First consequences are $\frac{\partial}{\partial t} \sim \mathbf{v}\cdot\nabla$ and $\frac{|\partial/\partial t|}{|\nabla|} \sim \mathcal{O}(1)$

Then the Lagrangian can be expressed as

$$\left(1 + g^{(2)}_{tt} + g^{(4)}_{tt} + 2g^{(3)}_{tm}v^m - \mathbf{v}^2 + g^{(2)}_{mn}v^m v^n\right)^{1/2}$$

and the metric tensor assumes the usual form

$$\begin{cases} g_{tt}(t,\mathbf{x}) \simeq 1 + g^{(2)}_{tt}(t,\mathbf{x}) + g^{(4)}_{tt}(t,\mathbf{x}) + \mathcal{O}(6) \\ g_{ti}(t,\mathbf{x}) \simeq g^{(3)}_{ti}(t,\mathbf{x}) + \mathcal{O}(5) \\ g_{ij}(t,\mathbf{x}) \simeq -\delta_{ij} + g^{(2)}_{ij}(t,\mathbf{x}) + \mathcal{O}(4) \end{cases}$$

For n = 2 we have the so-called Post-Newtonian Mechanics

The complete scheme of FOG at the newtonian limit

By using the Bianchi's identities and introducing two scale lengths m_1, m_2 the field equations in the newtonian limit (n = 1) are

$$\begin{cases} {m_1}^2 \doteq -\frac{f_X(0)}{3f_{XX}(0)+2f_Y(0)+2f_Z(0)} \\ {m_2}^2 \doteq \frac{f_X(0)}{f_Y(0)+4f_Z(0)]} \end{cases}$$

$$\begin{cases} R_{\alpha\mu\beta\nu;\delta}+R_{\alpha\mu\delta\beta;\nu}+R_{\alpha\mu\nu\delta;\beta}=0 \\ {R_{\alpha\mu\beta\nu}}^{;\alpha}+R_{\mu\beta;\nu}-R_{\mu\nu;\beta}=0 \\ 2{R_{\alpha\beta}}^{;\alpha}-R_{;\beta}=0 \\ 2{R_{\alpha\beta}}^{;\alpha\beta}-\Box R=0 \end{cases}$$

$$\begin{cases} (\triangle-{m_2}^2)R^{(2)}_{tt}+\left[\frac{{m_2}^2}{2}-\frac{{m_1}^2+2{m_2}^2}{6{m_1}^2}\triangle\right]X^{(2)} = -\frac{{m_2}^2\mathcal{X}}{f_X(0)}T^{(0)}_{tt} \\ (\triangle-{m_2}^2)R^{(2)}_{ij}+\left[\frac{{m_1}^2-{m_2}^2}{3{m_1}^2}\partial^2_{ij}-\left(\frac{{m_2}^2}{2}-\frac{{m_1}^2+2{m_2}^2}{6{m_1}^2}\triangle\right)\delta_{ij}\right]X^{(2)} = 0 \\ (\triangle-{m_1}^2)X^{(2)} = \frac{{m_1}^2\mathcal{X}}{f_X(0)}T^{(0)} \end{cases}$$

The general solution is found as follows

$$g^{(2)}_{tt}(t,\mathrm{x}) = \frac{1}{2\pi}\int d^3\mathrm{x}'d^3\mathrm{x}''\frac{\mathcal{G}_2(\mathrm{x}',\mathrm{x}'')}{|\mathrm{x}-\mathrm{x}'|}\left[\frac{{m_2}^2\mathcal{X}}{f_X(0)}T^{(0)}_{tt}(t,\mathrm{x}'')-\frac{({m_1}^2+2{m_2}^2)\mathcal{X}}{6f_X(0)}T^{(0)}t,\mathrm{x}'')+\frac{{m_2}^2-{m_1}^2}{6}X^{(2)}(t,\mathrm{x}'')\right]$$

$$X^{(2)}(t,\mathrm{x}) = \frac{{m_1}^2\mathcal{X}}{f_X(0)}\int d^3\mathrm{x}'\mathcal{G}_1(\mathrm{x},\mathrm{x}')T^{(0)}(t,\mathrm{x}') \qquad g^{(2)}_{ij} = -\frac{\delta_{ij}}{2\pi}\int d^3\mathbf{x}'d^3\mathbf{x}''\frac{\mathcal{G}_2(\mathbf{x}',\mathbf{x}'')}{|\mathbf{x}-\mathbf{x}'|}\left(\frac{{m_2}^2}{2}-\frac{{m_1}^2+2{m_2}^2}{6{m_1}^2}\triangle_{\mathbf{x}''}\right)X^{(2)}(\mathbf{x}'')$$

The “effective Lagrangian” of the FOG in the Newtonian level

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By using the Gauss – Bonnet invariant $\mathcal{G}_{GB} = X^2 - 4Y + Z$ we have the more simple but in the same time the more general FOG generating the previous solutions. At newtonian level any FOG collapses in an “effective lagrangian”.

$$\mathcal{A} = \int d^4x\sqrt{-g}\Big[\mathcal{L} + \mathcal{X}\mathcal{L}_m\Big] = \int d^4x\sqrt{-g}\Big[f(X,Y,Z) + \mathcal{X}\mathcal{L}_m\Big]$$

$$\mathcal{L} = f(X,Y,Z) = R - \frac{1}{3}\left[\frac{1}{2{\mu_1}^2} + \frac{1}{{\mu_2}^2}\right]R^2 + \frac{R_{\alpha\beta}R^{\alpha\beta}}{{\mu_2}^2}$$

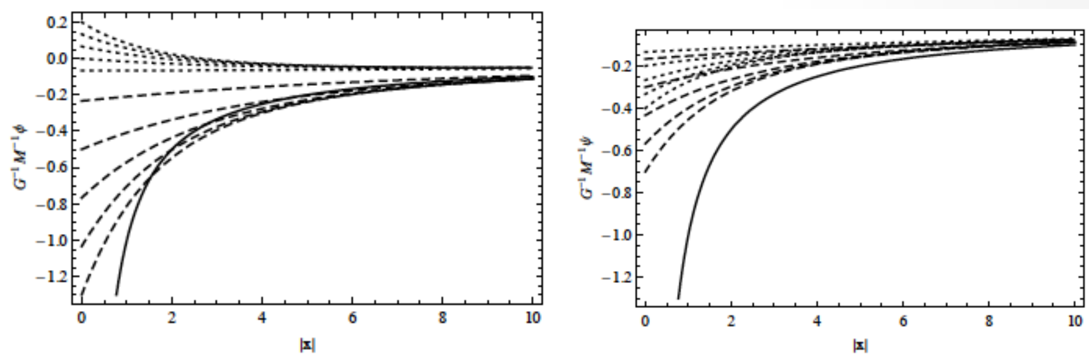
Identity in the field equation

$$H_{\mu\nu}^{GB} = H_{\mu\nu}^{X^2} - 4H_{\mu\nu}^{Y} + H_{\mu\nu}^{Z} = 0$$

In the case of pointlike source the solutions are

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Psi)\delta_{ij}dx^i dx^j$$

$$\begin{cases} X^{(2)} = -\dfrac{r_g\,{m_1}^2}{f_X(0)}\dfrac{e^{-m_1|\mathbf{x}|}}{|\mathbf{x}|} \\[2ex] \Phi = -\dfrac{GM}{f_X(0)}\left[\dfrac{1}{|\mathbf{x}|} + \dfrac{1}{3}\dfrac{e^{-m_1|\mathbf{x}|}}{|\mathbf{x}|} - \dfrac{4}{3}\dfrac{e^{-m_2|\mathbf{x}|}}{|\mathbf{x}|}\right] \\[2ex] \Psi = -\dfrac{GM}{f_X(0)}\left[\dfrac{1}{|\mathbf{x}|} - \dfrac{1}{3}\dfrac{e^{-m_1|\mathbf{x}|}}{|\mathbf{x}|} - \dfrac{2}{3}\dfrac{e^{-m_2|\mathbf{x}|}}{|\mathbf{x}|}\right] \end{cases}$$



For a right physical interpretation of potential we must have

$$f_{XX}(0) + f_Y(0) + 2f_Z(0) < 0$$

… while we have an extended mass …the Galaxy

Generally the superposition principle is not valid in FOG like in GR, but we are at newtonian level (linear field equations). Then we can assume

$$\Phi(\mathbf{x}) = -G\int d^3\mathbf{x}'\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\left[1+\frac{1}{3}e^{-\mu_1|\mathbf{x}-\mathbf{x}'|}-\frac{4}{3}e^{-\mu_2|\mathbf{x}-\mathbf{x}'|}\right]$$

and the galactic curve rotation

$$\frac{v_c(|\mathbf{x}|)^2}{|\mathbf{x}|} = \frac{\partial\Phi(\mathbf{x})}{\partial|\mathbf{x}|}$$

… and the potential is also gauge free!

There is a fundamental difference amoung the GR and FOG

validity or not of Gauss theorem

GR case

$$v_c(r)^2 = \frac{GM(r)}{r} = \frac{4\pi G}{r}\int_0^r dy\, y^2\,\rho(y)$$

FOG case

Only a numerical integration …

The galactic structure is a rotating system but the newtonian limit of the axially symmetry is the spherical one. The Galaxy is still.

In GR we have

$$g_{\mu\nu}^{Kerr} = \begin{pmatrix} 1-\frac{r_g r}{\Sigma^2} & 0 & 0 & \frac{r_g r\eta}{\Sigma^2}\sin^2\theta \\ 0 & -\frac{\Sigma^2}{H^2} & 0 & 0 \\ 0 & 0 & -\Sigma^2 & 0 \\ \frac{r_g r\eta}{\Sigma^2}\sin^2\theta & 0 & 0 & -\left(r^2+\eta^2+\frac{r_g r\eta^2}{\Sigma^2}\sin^2\theta\right)\sin^2\theta \end{pmatrix} \rightarrow \begin{pmatrix} 1-\frac{r_g}{r} & 0 & 0 & 0 \\ 0 & -1-\frac{r_g}{r} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

A simple model of Galaxy

Generally the potential is the sum of three components: bulge, disk and dark matter.

$$
\begin{cases}
\rho_{bulge}(r) = \dfrac{M_b}{2\pi\,\xi_b{}^{3-\gamma}\,\Gamma(\frac{3-\gamma}{2})}\,\dfrac{e^{-\frac{r^2}{\xi_b{}^2}}}{r^\gamma} \\
\sigma_{disk}(R) = \dfrac{M_d}{2\pi\,\xi_d{}^2}\,e^{-\frac{R}{\xi_d}} \\
\rho_{DM}(r) = \dfrac{\alpha\,M_{DM}}{\pi\,(4-\pi)\xi_{DM}{}^3}\,\dfrac{1}{1+\frac{r^2}{\xi_{DM}{}^2}}
\end{cases}
$$

$$
\begin{aligned}
\Phi(r,R,z) = \;& G\Bigg\{\frac{2\,M_b}{3\,\xi_b{}^{3-\gamma}\,\Gamma(\frac{3-\gamma}{2})}\frac{1}{r}\int_0^\infty dr'\,r'^{1-\gamma}\,e^{-\frac{r'^2}{\xi_b{}^2}}\Bigg[3\,\frac{|r-r'|-r-r'}{2} \\
& -\frac{e^{-\mu_1 a|r-r'|}-e^{-\mu_1 a(r+r')}}{2\,\mu_1\,a}+2\,\frac{e^{-\mu_2 a|r-r'|}-e^{-\mu_2 a(r+r')}}{\mu_2\,a}\Bigg] \\
& +\frac{4\,\alpha\,M_{DM}}{3(4-\pi)\,\xi_{DM}{}^3}\frac{1}{r}\int_0^{\Xi/a} dr'\,\frac{r'}{1+\frac{r'^2}{\xi_{DM}{}^2}}\Bigg[3\,\frac{|r-r'|-r-r'}{2} \\
& -\frac{e^{-\mu_1 a|r-r'|}-e^{-\mu_1 a(r+r')}}{2\,\mu_1\,a}+2\,\frac{e^{-\mu_2 a|r-r'|}-e^{-\mu_2 a(r+r')}}{\mu_2\,a}\Bigg] \\
& -\frac{M_d}{\pi\,\xi_d{}^2}\Bigg[\int_0^\infty dR'\,e^{-\frac{R'}{\xi_d}}\,R'\left(\frac{\mathfrak{K}(\frac{4RR'}{(R+R')^2+z^2})}{\sqrt{(R+R')^2+z^2}}+\frac{\mathfrak{K}(\frac{-4RR'}{(R-R')^2+z^2})}{\sqrt{(R-R')^2+z^2}}\right) \\
& +\int_0^\infty dR'\,e^{-\frac{R'}{\xi_d}}\,R'\int_0^\pi d\theta'\,\frac{e^{-\mu_1 a\,\Delta(R,R',z,0,\theta')}-4\,e^{-\mu_2 a\,\Delta(R,R',z,0,\theta')}}{3\,\Delta(R,R',z,0,\theta')}\Bigg]\Bigg\}
\end{aligned}
$$

r: radial distance from the galactic center;

(R, θ, z): cylindrical coordinates; $r = \sqrt{R^2+z^2}$

(M_b, M_d, M_{DM}): masses of the components;

(ξ_b, ξ_d, ξ_{DM}): effective radii;

α: ratio of Dark Matter inside the sphere with radius ξ_{DM} with respect to the total one;

γ: fitting parameter

Ξ: observing distance of rotation curve

a: unit of distance (1 Kpc)

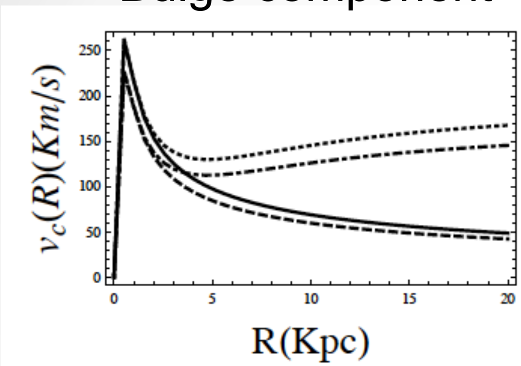
$\mathfrak{K}$: elliptic function

$$\Delta(R,R',z,z',\theta') \doteq |\mathbf{x}-\mathbf{x}'| = \sqrt{(R+R')^2+(z-z')^2-4RR'\cos^2\theta'}$$

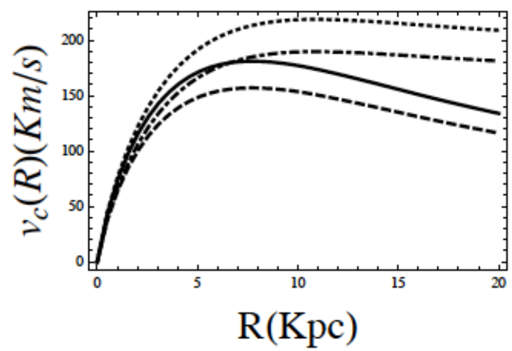
The rotation velocity $$v_c(R) = \sqrt{R\,\frac{\partial}{\partial R}\Phi(R,R,0)}$$

Theoretical and experimental rotation curves

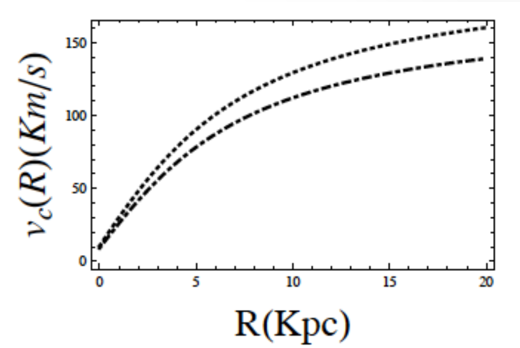
Bulge component



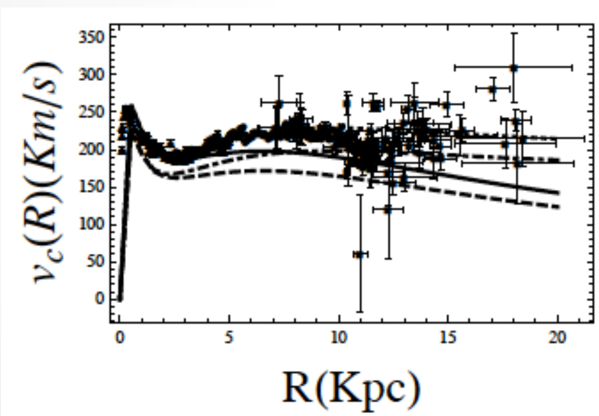
Disk component



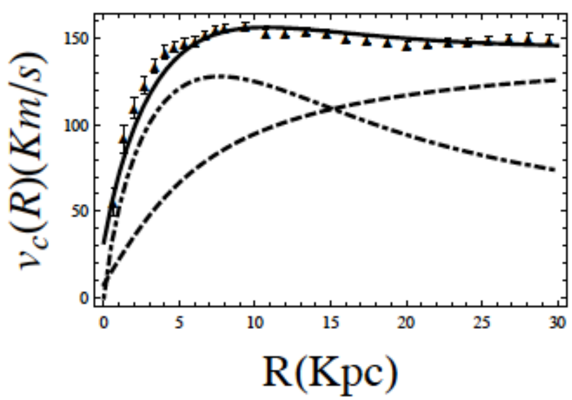
DM component



Milky Way



NGC 3198



GR: Dashed line

GR + DM: Dashed and dotted line

FOG: Solid line

FOG + DM: Dotted line

$\mu_1 = 10^{-2}$ Kpc^{-1}

$\mu_2 = 10^{2}$ Kpc^{-1}

Galaxy	M_b	ξ_b	γ	M_d	ξ_d	M_{DM}	ξ_{DM}	α	Ξ
Milky Way	0.77	0.5	1.5	5.20	3.5	1.68	5.5	0.50	20
NGC 3198	0	/	/	2.60	3.5	0.84	5.5	0.53	20

The unity of mass is 10^{10} solar mass;
The unity of distance is 1 Kpc.

We need DM component!

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Gravitational Lensing by extended matter source

The starting point is the the metric

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Psi)\delta_{ij}dx^i dx^j$$

$$\begin{cases} \Phi = -G\int d^3\mathbf{x}' \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\left[1+\frac{1}{3}e^{-\mu_1|\mathbf{x}-\mathbf{x}'|}-\frac{4}{3}e^{-\mu_2|\mathbf{x}-\mathbf{x}'|}\right] \\ \Psi = -G\int d^3\mathbf{x}' \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\left[1-\frac{1}{3}e^{-\mu_1|\mathbf{x}-\mathbf{x}'|}-\frac{2}{3}e^{-\mu_2|\mathbf{x}-\mathbf{x}'|}\right] \end{cases}$$

From the relativistic distance $ds^2 = 0$ $g_{\alpha\beta}u^\alpha u^\beta = (1+2\Phi){u^0}^2 - (1-2\Psi)|\mathbf{u}|^2 = 0$ we find $u^\mu = \left(\sqrt{\frac{1-2\Psi}{1+2\Phi}}|\mathbf{u}|, \mathbf{u}\right)$

and the newtonian limit of the geodesic motion $\dot{u}^\mu + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta = 0 \rightarrow \quad \dot{\mathbf{u}} = -2\left[\nabla_\perp\Psi + \frac{1}{2}\nabla(\Phi-\Psi)\right]$

If $\Phi = \Psi$ the outcome of GR is recovered.

$$\nabla_\perp = \nabla - \left(\frac{\mathbf{u}}{|\mathbf{u}|}\cdot\nabla\right)\frac{\mathbf{u}}{|\mathbf{u}|}$$

But in FOG we have …

$$\Delta(\Phi-\Psi) = \frac{{m_1}^2 - {m_2}^2}{3{m_1}^2}\int d^3\mathbf{x}'\mathcal{G}_2(\mathbf{x},\mathbf{x}')\,\Delta_{\mathbf{x}'}X^{(2)}(\mathbf{x}')$$

The "vectorial" deflection angle is defined by $\vec{\alpha} = -\int_{\lambda_i}^{\lambda_f}\frac{d\mathbf{u}}{d\lambda}d\lambda = \int_{z_i}^{z_f}\left[\nabla_{\vec{\xi}}(\Phi+\Psi) + \hat{z}\,\partial_z(\Phi-\Psi)\right]dz$

$$\begin{aligned}\vec{\alpha} = {} & 2G\int_{z_i}^{z_f} d^2\vec{\xi}'dz'\,dz\frac{\rho(\vec{\xi}',z')(\vec{\xi}-\vec{\xi}')}{\Delta(\vec{\xi},\vec{\xi}',z,z')^3} - 2G\int_{z_i}^{z_f} d^2\vec{\xi}'dz'\,dz\frac{\rho(\vec{\xi}',z')[1+\mu_2\Delta(\vec{\xi},\vec{\xi}',z,z')]}{\Delta(\vec{\xi},\vec{\xi}',z,z')^3}e^{-\mu_2\Delta(\vec{\xi},\vec{\xi}',z,z')}(\vec{\xi}-\vec{\xi}') \\ & + \frac{2G}{3}\hat{z}\int_{z_i}^{z_f} d^2\vec{\xi}'dz'\,dz\frac{\rho(\vec{\xi}',z')(z-z')}{\Delta(\vec{\xi},\vec{\xi}',z,z')^3}\left[\left(1+\mu_1\Delta(\vec{\xi},\vec{\xi}',z,z')\right)e^{-\mu_1\Delta(\vec{\xi},\vec{\xi}',z,z')} - \left(1+\mu_2\Delta(\vec{\xi},\vec{\xi}',z,z')\right)e^{-\mu_2\Delta(\vec{\xi},\vec{\xi}',z,z')}\right]\end{aligned} \quad (34)$$

… in the limit of planar and point-like source

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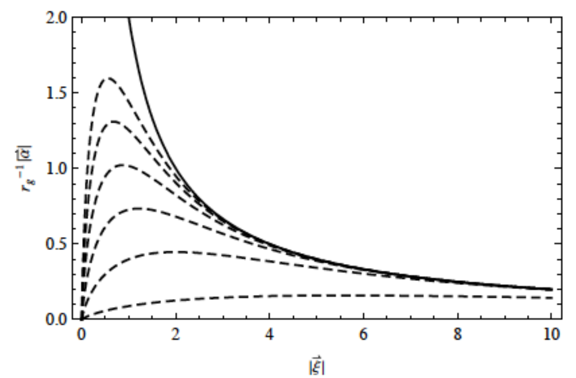
Two-dimensional density $\Sigma(\vec\xi') = \int dz'\rho(\vec\xi',z')$

Deflection angle $\vec\alpha = 4G\int d^2\vec\xi'\,\Sigma(\vec\xi')\left[\frac{1}{|\vec\xi-\vec\xi'|} - |\vec\xi-\vec\xi'|\,\mathcal{F}_{\mu_2}(\vec\xi,\vec\xi')\right]\frac{\vec\xi-\vec\xi'}{|\vec\xi-\vec\xi'|}$ $\quad \mathcal{F}_{\mu_2}(\vec\xi,\vec\xi') = \int_0^\infty dz\,\frac{(1+\mu_2\Delta(\vec\xi,\vec\xi',z,0))}{\Delta(\vec\xi,\vec\xi',z,0)^3}e^{-\mu_2\Delta(\vec\xi,\vec\xi',z,0)}$

and in the limit of point-like source $\vec\alpha = 2r_g\left[\frac{1}{|\vec\xi|} - |\vec\xi|\,\mathcal{F}_{\mu_2}(\vec\xi,0)\right]\frac{\vec\xi}{|\vec\xi|}$



If $f(X,Y,Z) \to f(X)$ $(\mu_2 \to \infty)$ $\mathcal{F}_{\mu_2}(\vec\xi,\vec\xi') \to 0$

The correction to the outcome of GR is f(X)-independent

***f*(*R*)-Gravity admits the same results of GR.**

In the limit of weak field ($r_g/r \ll 1$) $b \approx r_0$, then

$$\alpha = 2r_g\left[\frac{1}{b} - b\,\mathcal{F}_{\mu_2}(b,0)\right] + \beta = \theta - \frac{D_{LS}}{D_{OS}}\alpha = [1+\theta_E{}^2\mathcal{F}(\theta)]\,\theta^2 - \beta\,\theta - \theta_E{}^2 = 0 \qquad \mathcal{F}(\theta) = \int_0^\infty dz\,\frac{(1+\mu_2 D_{OL}\sqrt{\theta^2+z^2})}{\sqrt{(\theta^2+z^2)^3}}e^{-\mu_2 D_{OL}\sqrt{\theta^2+z^2}}$$

(lens equation)

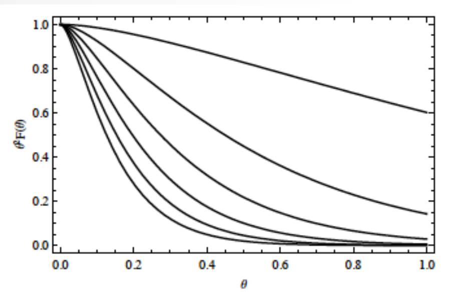
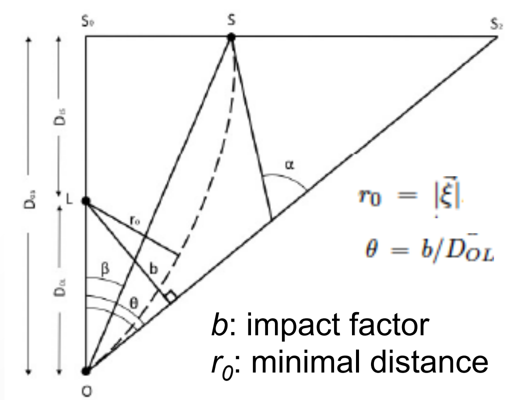


Image positions $\theta = \theta_\pm^{GR} \mp \frac{\theta_E{}^2}{\sqrt{\beta^2+4\theta_E{}^2}}\mathcal{F}(\theta_\pm^{GR})\,\theta_\pm^{GR\,2}$

In the case $\beta = 0$ $\quad \theta = \pm\theta_E\left[1 - \frac{\theta_E{}^2}{2}\mathcal{F}(\theta_E)\right]$

Correction to the Einstein ring



Conclusions

1. Any $f(R)$-Gravity contributes to the more attractive force, but the Ricci tensor invariant contributes to the anti-Gravity: we have a lower theoretical rotation velocity (with respect to the pure $f(R)$-Gravity), while the experimental evidence says opposite.

2. From the point of view of gravitational lensing we have a perfect agreement with the GR. Only by adding $f(R_{ab}\,R^{ab})$ in the action we induce the modifications, but we do not find the hoped behavior.

3. In the galactic dynamics we are studying the motion of massive particles and in this case we find the corrections induced also by only $f(R)$-Gravity.

4. The galactic rotation curves need a Dark Matter component, but the choice of framework is crucial for the space model of Dark Matter.

5. Moreover if we consider $f(R,\,R_{ab}\,R^{ab})$-Gravity for the gravitational lensing we need a bigger amount of Dark Matter then in GR.

6. The behavior of anti-Gravity is similar to the Sanders potential … but that's an another history

7. Dark matter effect as a single geometric phenomenon remains a hard challenge.

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