Experimental tests of Extended Theories of Gravity

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- Scalar Tensor Fourth Order Gravity (STFOG)

References

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The action

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R, R_{\alpha\beta}R^{\alpha\beta}, \phi) + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha} + \mathcal{X}\mathcal{L}_{matter}\right]$$

- $R \rightarrow$ Ricci scalar
- $R_{\alpha\beta}R^{\alpha\beta} \doteq Y \rightarrow$Ricci tensor square
- $\phi \rightarrow$ scalar field
- $\mathcal{L}_{matter} \rightarrow$ minimally coupled ordinary matter
- $g \rightarrow$ determinant of metric tensor $g_{\mu\nu}$
- $\mathcal{X} = 8\pi G/c^4$ but we use $c = 1$
- $R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$, $R^{\alpha}{}_{\beta\mu\nu} = \Gamma^{\alpha}_{\beta\nu,\mu} + \ldots \rightarrow$ convention
- $2\Gamma^{\mu}_{\alpha\beta} = g^{\mu\sigma}(g_{\alpha\sigma,\beta} + g_{\beta\sigma,\alpha} - g_{\alpha\beta,\sigma})$
- $(+ - - -) \rightarrow$ the adopted signature

Scalar Tensor Fourth Order Gravity

The field equations by applying $\delta(\cdot) \rightarrow \delta g_{\mu\nu}\frac{\delta(\cdot)}{\delta g_{\mu\nu}} + \delta\phi\frac{\delta(\cdot)}{\delta\phi}$

$$f_R R_{\mu\nu} - \frac{f + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha}}{2} g_{\mu\nu} - f_{R;\mu\nu} + g_{\mu\nu}\Box f_R + 2 f_Y {R_\mu}^\alpha R_{\alpha\nu} - 2[f_Y {R^\alpha}_{(\mu}]_{;\nu)\alpha}$$
$$+\Box[f_Y R_{\mu\nu}] + [f_Y R_{\alpha\beta}]^{;\alpha\beta} g_{\mu\nu} + \omega(\phi)\phi_{;\mu}\phi_{;\nu} = \mathcal{X}\, T_{\mu\nu}$$

$$2\omega(\phi)\Box\phi + \omega_\phi(\phi)\phi_{;\alpha}\phi^{;\alpha} - f_\phi = 0$$

$$f_R R + 2 f_Y R_{\alpha\beta}R^{\alpha\beta} - 2f + \Box[3 f_R + f_Y R] + 2[f_Y R^{\alpha\beta}]_{;\alpha\beta} - \omega(\phi)\phi_{;\alpha}\phi^{;\alpha} = \mathcal{X}\, T$$

- $T_{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$ → the tensor of matter
- $T = {T^\rho}_\rho$ → trace of matter tensor
- $f_R = \frac{df}{dR}$, $f_Y = \frac{df}{dY}$, $\omega_\phi = \frac{d\omega}{d\phi}$, $f_\phi = \frac{df}{d\phi}$, $\Box = {}_{;\sigma}{}^{;\sigma}$

The scalar field ϕ is coupled with the geometry R, $R_{\mu\nu}$ (as the classical scalar tensor gravity) and the field equations are differential equations of fourth order.

The first two interesting frameworks are

Newtonian and Post-Newtonian limit	Weak field limit
$g_{\mu\nu} \sim \begin{pmatrix} 1 + g^{(2)}_{tt}(t,\mathbf{x}) + g^{(4)}_{tt}(t,\mathbf{x}) & g^{(3)}_{ti}(t,\mathbf{x}) \\ g^{(3)}_{ti}(t,\mathbf{x}) & -\delta_{ij} + g^{(2)}_{ij}(t,\mathbf{x}) \end{pmatrix}$	$g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu}(x)$
$\phi \sim \phi^{(0)} + \phi^{(2)}(t,\mathbf{x})$	$\phi \sim \phi^{(0)} + \phi^{(1)}(x)$
$;\mu \sim ,\mu + \tilde{;}\mu = -\nabla + \tilde{\partial}_\mu \qquad \Box \sim -\triangle + \tilde{\Box}$	$;\mu \sim ,\mu = \partial_\mu \qquad \Box \sim \Box_\eta$
Slow motion in the spherically symmetric systems	Gravitational waves

- $x = (t, \mathbf{x})$
- The approximation level is driven by an expansion of the powers of $1/c$
- The approximation level is driven by the linearization of field equations $\mathcal{O}(h_{\mu\nu})^2 \ll 1$
- In the Newtonian limit the space time is time independet

The Newtonian limit of STFOG

By setting $\omega(\phi^{(0)}) = 1/2$ the field equations are

$$(\triangle - m_Y{}^2)\triangle\Phi + \left[\frac{m_Y{}^2}{2} - \frac{m_R{}^2+2m_Y{}^2}{6m_R{}^2}\triangle\right]R + m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\triangle\varphi = -m_Y{}^2\mathcal{X}\,T_{tt}$$

$$(\triangle - m_R{}^2)R - 3m_R{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\triangle\varphi = m_R{}^2\,\mathcal{X}\,T$$

$$(\triangle - m_\phi{}^2)\varphi + f_{R\phi}(0,0,\phi^{(0)})\,R = 0$$

where

$$g_{\mu\nu} \sim \begin{pmatrix} 1+2\Phi(\mathbf{x}) & 0 \\ 0 & -\delta_{ij} \end{pmatrix}$$

$$\phi \sim \phi^{(0)} + \varphi$$

The derivatives are calculated on the Minkowskian background

$$m_R{}^2 \doteq -\frac{f_R(0,0,\phi^{(0)})}{3f_{RR}(0,0,\phi^{(0)})+2f_Y(0,0,\phi^{(0)})}$$

$$m_Y{}^2 \doteq \frac{f_R(0,0,\phi^{(0)})}{f_Y(0,0,\phi^{(0)})}$$

$$m_\phi{}^2 \doteq -f_{\phi\phi}(0,0,\phi^{(0)})$$

- $T_{tt} = T = \rho(\mathbf{x}) \rightarrow$ the energy mass density of the source
- $T = T^\rho{}_\rho \rightarrow$ trace of matter tensor
- **The theory is parameterized by the coefficients of Taylor expansion of Lagrangian.**

The Newtonian limit of STFOG: the pointlike gravitational potential

The scalar field φ and also the Ricci scalar R are auxiliary fields. The physical outcome is

$$\Phi(\mathbf{x}) = -\frac{GM}{|\mathbf{x}|}\left[1 + g(\xi,\eta)\,e^{-m_+|\mathbf{x}|} + [1/3 - g(\xi,\eta)]\,e^{-m_-|\mathbf{x}|} - \frac{4}{3}\,e^{-m_Y|\mathbf{x}|}\right]$$

where $g(\xi,\eta) = \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$, $w_\pm = \frac{1-\xi+\eta^2\pm\sqrt{(1-\xi+\eta^2)^2-4\eta^2}}{2}$, $\eta = \frac{m_\phi}{m_R}$,

$m_\pm^2 = m_R^2 w_\pm$ and $\xi = 3f_{R\phi}(0,0,\phi^{(0)})^2$.

The effective Lagrangian of STFOG is given as

$$R + \frac{f_{RR}(0,0,\phi^{(0)})}{2}\,R^2 + \frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2}\,\varphi^2 + f_{R\phi}(0,0,\phi^{(0)})R\,\phi + f_Y(0,0,\phi^{(0)})R_{\alpha\beta}R^{\alpha\beta} + \frac{|\nabla\varphi|^2}{2}$$

. . . and if we add the terms proportional to $R_{\alpha\beta\gamma\beta}R^{\alpha\beta\gamma\delta}$ and/or to $\Box^k R$? **The outcome is the same**!!

- The Gauss-Bonnet invariant . . .
- $\Box^k$ is the four divergence . . .

The Newtonian limit of STFOG: Classes of the gravitational potentials (A, B)

Case	Gravitational potential	Free parameters
A	$-\frac{GM}{\vert\mathbf{x}\vert}\left[1+\frac{1}{3}\,e^{-m_R\vert\mathbf{x}\vert}\right]$	${m_R}^2 = -\frac{f_R(0)}{3f_{RR}(0)}$
B	$-\frac{GM}{\vert\mathbf{x}\vert}\left[1+\frac{1}{3}\,e^{-m_R\vert\mathbf{x}\vert}-\frac{4}{3}\,e^{-m_Y\vert\mathbf{x}\vert}\right]$	${m_R}^2 = -\frac{f_R(0,0)}{3f_{RR}(0,0)+2f_Y(0,0)}$ ${m_Y}^2 = \frac{f_R(0,0)}{f_Y(0,0)}$

- case A $\rightarrow f(R)$
- case B $\rightarrow f(R,\ R_{\alpha\beta}R^{\alpha\beta})$
- case C $\rightarrow f(R,\ \phi)+\omega(\phi)\phi_{;\alpha}\phi^{;\alpha}$
- case D $\rightarrow f(R,\ R_{\alpha\beta}R^{\alpha\beta},\ \phi)+\omega(\phi)\phi_{;\alpha}\phi^{;\alpha}$

The Newtonian limit of STFOG: Classes of the gravitational potentials (C)

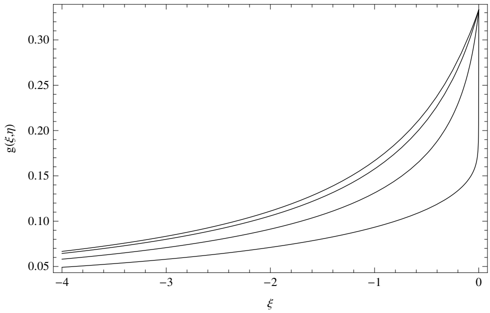
Gravitational potential	Free parameters
$-\frac{GM}{\lvert\mathbf{x}\rvert}\Big[1+g(\xi,\eta)\,e^{-m_+\,\lvert\mathbf{x}\rvert}+ +[1/3-g(\xi,\eta)]\,e^{-m_-\,\lvert\mathbf{x}\rvert}\Big]$	$m_R{}^2 = -\frac{f_R(0,\phi^{(0)})}{3f_{RR}(0,\phi^{(0)})}$ $m_\phi{}^2 = -\frac{f_{\phi\phi}(0,\phi^{(0)})}{2\omega(\phi^{(0)})}$ $\xi = \frac{3f_{R\phi}(0,\phi^{(0)})^2}{2\omega(\phi^{(0)})}$ $\eta = \frac{m_\phi}{m_R}$ $g(\xi,\,\eta) = \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$ $w_\pm = \frac{1-\xi+\eta^2\pm\sqrt{(1-\xi+\eta^2)^2-4\eta^2}}{2}$

The Newtonian limit of STFOG: Classes of the gravitational potentials (D)

Gravitational potential	Free parameters
$-\frac{GM}{\vert\mathbf{x}\vert}\Big[1+g(\xi,\,\eta)\,e^{-m_+\,\vert\mathbf{x}\vert}+ +[1/3-g(\xi,\,\eta)]\,e^{-m_-\,\vert\mathbf{x}\vert}-\frac{4}{3}\,e^{-m_Y\vert\mathbf{x}\vert}\Big]$	${m_R}^2 = -\frac{f_R(0,0,\phi^{(0)})}{3f_{RR}(0,0,\phi^{(0)})+2f_Y(0,0,\phi^{(0)})}$ ${m_Y}^2 = \frac{f_R(0,0,\phi^{(0)})}{f_Y(0,0,\phi^{(0)})}$ ${m_\phi}^2 = -\frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2\omega(\phi^{(0)})}$ $\xi = \frac{3{f_{R\phi}(0,0,\phi^{(0)})}^2}{2\omega(\phi^{(0)})}$ $\eta = \frac{m_\phi}{m_R}$ $g(\xi,\,\eta) =$ $= \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$ $w_\pm = \frac{1-\xi+\eta^2\pm\sqrt{(1-\xi+\eta^2)^2-4\eta^2}}{2}$

The Newtonian limit of STFOG: The crucial choice of the coefficient and of the theory!

The potential does not depend only on the masses propagating (cases A, B) but also on the function modeling the Yukawa corrections (case C, D).



Plot of coefficient $g(\xi,\eta)$ (case C) with respect to quantity ξ for $0 \leq \eta \leq 0.99$ with step 0.33.

$g(\xi,\eta) = \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$, $w_\pm = \frac{1-\xi+\eta^2\pm\sqrt{(1-\xi+\eta^2)^2-4\eta^2}}{2}$, $\eta = \frac{m_\phi}{m_R}$ and $\xi = \frac{3f_{R\phi}(0,0,\phi^{(0)})^2}{2\omega(\phi^{(0)})}$.

The Newtonian limit of STFOG: the Gauss theorem is not applicable!

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- The case of source ball-like is different from the pointlike one: the potential depends on the dimension of the source.

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- Generally for any term $\propto \frac{e^{-mr}}{r}$ we have a geometric factor multipliyng the Yukawa term.

The Newtonian limit of STFOG: the Gauss theorem is not applicable!

- **In STFOG the Gauss theorem is not applicable!**
- The case of source ball-like is different from the pointlike one: the potential depends on the dimension of the source.
- Generally for any term $\propto \frac{e^{-mr}}{r}$ we have a geometric factor multipliyng the Yukawa term.
- In the case of a ball with radius $\mathcal{R}$ we find $F(m\mathcal{R}) = 3\frac{m\mathcal{R}\cosh m\mathcal{R} - \sinh m\mathcal{R}}{m^3\mathcal{R}^3}$

$$\Phi_{ball}(\mathbf{x}) = -\frac{GM}{|\mathbf{x}|}\left[1 + g(\xi,\eta)\,F(m_+\mathcal{R})\,e^{-m_+|\mathbf{x}|} + [1/3 - g(\xi,\eta)]\,F(m_-\mathcal{R})\,e^{-m_-|\mathbf{x}|} - \frac{4}{3}\,F(m_Y\mathcal{R})\,e^{-m_Y|\mathbf{x}|}\right]$$

The Post-Newtonian limit of STFOG

- Post-Newtonian field equations:

$$\left\{(\triangle - m_Y{}^2)\triangle\Psi - \left[\frac{m_Y{}^2}{2} - \frac{m_R{}^2+2m_Y{}^2}{6m_R{}^2}\triangle\right]R - m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\triangle\varphi\right\}\delta_{ij}$$

$$+\left\{(\triangle - m_Y{}^2)(\Psi-\Phi) + \frac{m_R{}^2 - m_Y{}^2}{3m_R{}^2}\,R + m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\varphi\right\}_{,ij} = 0\,,$$

$$\left\{(\triangle - m_Y{}^2)\triangle A_i + m_Y{}^2\mathcal{X}\,\rho\, v_i\right\} + \left\{(\triangle - m_Y{}^2)\Psi + \frac{m_R{}^2 - m_Y{}^2}{3m_R{}^2}R + m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\varphi\right\}_{,ti}$$

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$$+\left\{(\triangle - m_Y{}^2)(\Psi - \Phi) + \frac{m_R{}^2 - m_Y{}^2}{3m_R{}^2}\,R + m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\varphi\right\}_{,ij} = 0\,,$$

$$\left\{(\triangle - m_Y{}^2)\triangle A_i + m_Y{}^2\mathcal{X}\,\rho\, v_i\right\} + \left\{(\triangle - m_Y{}^2)\Psi + \frac{m_R{}^2 - m_Y{}^2}{3m_R{}^2}R + m_Y{}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\varphi\right\}_{,ti}$$

- The solutions for a ball with radius $\mathcal{R}$ are:

$$\Psi_{ball}(\mathbf{x}) = -\frac{GM}{|\mathbf{x}|}\Big[1 - g(\xi,\eta)\,F(m_+\mathcal{R})\,e^{-m_+|\mathbf{x}|} - \frac{2\,F(m_Y\mathcal{R})}{3}\,e^{-m_Y|\mathbf{x}|} + -[\frac{1}{3} - g(\xi,\eta)]\,F(m_-\mathcal{R})\,e^{-m_-|\mathbf{x}|}\Big]\,.$$

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1 - (1 + m_Y|\mathbf{x}|)\,e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}}\times\mathbf{J}\,,$$

summarizing

- The metric is:

$$
\begin{aligned}
g_{tt} &= 1 + 2\Phi_{ball}(\mathbf{x}) \\
g_{ti} &= 2A_i(\mathbf{x}) \\
g_{ij} &= -\delta_{ij} + 2\Psi_{ball}(\mathbf{x})\delta_{ij}
\end{aligned}
$$

summarizing

- The metric is:

$$\begin{aligned} g_{tt} &= 1 + 2\Phi_{ball}(\mathbf{x}) \\ g_{ti} &= 2A_i(\mathbf{x}) \\ g_{ij} &= -\delta_{ij} + 2\Psi_{ball}(\mathbf{x})\delta_{ij} \end{aligned}$$

- The non-vanishing Christoffel symbols are:

$$\begin{aligned} \Gamma^t_{ti} &= \Gamma^i_{tt} = \partial_i \Phi_{\mathrm{ball}} \ , \\ \Gamma^i_{tj} &= \frac{\partial_i A_j - \partial_j A_i}{2} \ , \\ \Gamma^i_{jk} &= \delta_{jk}\partial_i \Psi_{\mathrm{ball}} - \delta_{ij}\partial_k \Psi_{ball} - \delta_{ik}\partial_j \Psi_{\mathrm{ball}} \ . \end{aligned}$$

Rotating sources and orbital parameters

- Let us consider the geodesic equations:

$$\frac{d^2\,x^\mu}{ds^2}+\Gamma^\mu_{\alpha\beta}\,\frac{dx^\alpha}{ds}\,\frac{dx^\beta}{ds}\;=\;0$$

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- We get the accelerations:

$$\ddot{x}+\frac{GM}{r^3}x\;=\;-\frac{GM\Lambda(r)}{r^3}x+\frac{2GJ}{r^5}\left\{\zeta(r)\left[\left(x^2+y^2-2z^2\right)\dot{y}+3yz\dot{z}\right]+2\Sigma(r)L_x z\right\}$$

$$\ddot{y}+\frac{GM}{r^3}y\;=\;-\frac{GM\Lambda(r)}{r^3}y-\frac{2GJ}{r^5}\left\{\zeta(r)\left[\left(x^2+y^2-2z^2\right)\dot{x}+3xz\dot{z}\right]-2\Sigma(r)L_y z\right\}$$

$$\ddot{z}+\frac{GM}{r^3}z\;=\;-\frac{GM\Lambda(r)}{r^3}z+\frac{6GJ}{r^5}\left\{\zeta(r)+\frac{2}{3}\Sigma(r)\right\}L_z z$$

Rotating sources and orbital parameters

- Let us consider the geodesic equations:

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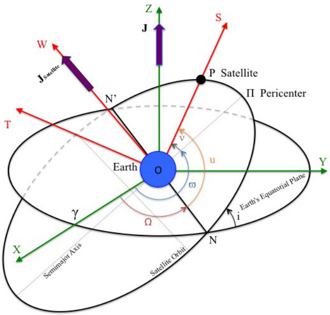
\ddot{z} + \frac{GM}{r^3}z &= -\frac{GM\Lambda(r)}{r^3}z + \frac{6GJ}{r^5}\left\{\zeta(r) + \frac{2}{3}\Sigma(r)\right\}L_z z
\end{aligned}$$

- Where L_x,L_y and L_z are the components of the angular momentum and...

$$\begin{aligned}
\Lambda(r) &\doteq g(\xi,\eta)\,F(m_+\mathcal{R})\,(1+m_+r)\,e^{-m_+r} - \frac{4\,F(m_Y\mathcal{R})}{3}\,(1+m_Yr)\,e^{-m_Yr} +\\
&\quad +[1/3 - g(\xi,\eta)]\,F(m_-\mathcal{R})\,(1+m_-r)\,e^{-m_-r}\\
\zeta(r) &\doteq 1 - \left[1 + m_Y r + (m_Y r)^2\right]e^{-m_Y r}\\
\Sigma(r) &\doteq (m_Y r)^2 e^{-m_Y r}
\end{aligned}$$

Orbital parameters



Is useful to introduce the conventional astronomical notation: a semimajor axis; e eccentricity; $p = a(1 - e^2)$ semilatus rectum; i inclination; Ω longitude of the ascending node N; $\tilde{\omega}$ longitude of the pericenter Π; L_0, mean longitude of the epoch, i.e. longitude of the satellite at time $t = 0$ (also a broken angle, measured from the X axis); ν true anomaly; $u = \nu + \tilde{\omega} - \Omega$ argument of the latitude; $n = (GM/a^3)^{1/2}$ mean daily motion; $C = r^2\dot{\nu} = na^2(1 - e^2)^{1/2}$ twice the areal velocity.

Gauss equations

- The Gauss equations for the variations of the six orbital parameters, resulting from the perturbing acceleration with components A_x, A_y, A_z, read:

$$\frac{da}{dt} = \dot{a}_{\rm STFOG} = \frac{2e\, G\, M\, \Lambda(r)\, \sin\nu}{n\sqrt{1-e^2}\, C}\,\dot{\nu}$$

$$\frac{de}{dt} = \dot{e}_{\rm GR} + \dot{e}_{\rm STFOG} = \frac{\sqrt{1-e^2}\, G\, M\, \Lambda(r)\, \sin\nu}{n\, a\, C}\,\dot{\nu} + \dot{e}_{\rm GR}\left[1 - e^{-m_Y r}\left(1 + m_Y r + (m_Y r)^2\right)\right]$$

$$\frac{d\Omega}{dt} = \dot{\Omega}_{\rm GR} + \dot{\Omega}_{\rm STFOG} = \dot{\Omega}_{\rm GR}\left\{1 - e^{-m_Y r}\left[1 + m_Y r + \left(1 + f(\nu, u, e)\right)(m_Y r)^2\right]\right\}$$

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$$\frac{d\tilde{\omega}}{dt} = \dot{\tilde{\omega}}_{\rm GR} + \dot{\tilde{\omega}}_{\rm STFOG} = -\frac{\sqrt{1-e^2}\, G\, M\, \Lambda(r)\, \cos\nu}{n\, a\, e\, C}\,\dot{\nu} + \dot{\tilde{\omega}}_{\rm GR}\left[1 - e^{-m_Y r}\left(1 + m_Y r + (m_Y r)^2\right)\right] + -2\sin^2\frac{i}{2}\,\dot{\Omega}_{\rm GR} f(\nu, u, e)\Sigma(r)$$

$$\frac{dM^0}{dt} = \dot{M}^0{}_{\rm GR} + \dot{M}^0{}_{\rm STFOG} = -\frac{G\, M\, \Lambda(r)}{n\, a\, C}\left[\frac{2r}{a} + \frac{e\sqrt{1-e^2}}{1+\sqrt{1-e^2}}\cos\nu\right]\dot{\nu} + +\dot{M}^0{}_{\rm GR}\left[1 - e^{-m_Y r}\left(1 + m_Y r + (m_Y r)^2\right)\right] - 2\sin^2\frac{i}{2}\,\dot{\Omega}_{\rm GR} f(\nu, u, e)\Sigma(r)$$

- where:

$$f(\nu, u, e) = \frac{1 + e\cos\nu}{1 + e\left(\frac{\sin\nu\,\cot u}{2} + \cos\nu\right)}$$

Gauss equations: secular terms

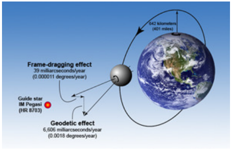
- Considering an almost circular orbit ($e \ll 1$), we integrate the Gauss equations with respect to the only anomaly ν, from 0 to $\nu(t) = nt$, since all other parameters have a slower evolution than ν, hence they can be considered as constraints with respect to ν. At first order we get

$$
\begin{aligned}
\Delta a(t) &= 0 \\
\Delta e(t) &= 0 \\
\Delta i(t) &= \frac{G\,J\,e^2\,\sin i}{na^3}\,e^{-m_Y p}(m_Y p)^2\left[1+\frac{(m_Y p)^2}{2}\left(m_Y p-4\right)\right]\sin\left(\tilde{\omega}(t)-\Omega(t)\right)\nu(t)+\mathcal{O}(e^4) \\
\Delta\Omega(t) &= \frac{2\,G\,J}{na^3}\left[1-e^{-m_Y p}\left(1+m_Y p+2(m_Y p)^2\right)\right]\nu(t)+\mathcal{O}(e^2) \\
\Delta\tilde{\omega}(t) &= \left\{\frac{\tilde{\Lambda}(p)}{2}-\frac{2\,G\,J}{na^3}\left[3\cos i-1+e^{-m_Y p}(1+m_Y p+\tfrac{3}{2}(m_Y p)^2+\right.\right. \\
&\qquad \left.\left.-(3+3m_Y p+3(m_Y p)^2+\tfrac{1}{12}(m_Y p)^3)\cos i)\right]\right\}\nu(t)+\mathcal{O}(e^2) \\
\Delta\mathcal{M}^0(t) &= \left\{2\Lambda(p)-\frac{2\,G\,J}{na^3}\left[3\cos i-1-e^{-m_Y p}\left(1+m_Y p+2(m_Y p)^2\right)\left(\cos i-1\right)\right]\right\}\nu(t)+\mathcal{O}(e^2)
\end{aligned}
$$

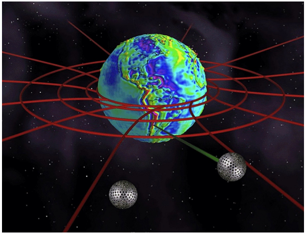
where

$$
\begin{aligned}
\tilde{\Lambda}(p) \doteq\; & g(\xi,\eta)\,F(m_+\mathcal{R})\,(m_+p)^2\,e^{-m_+p}+[1/3-g(\xi,\eta)]\,F(m_-\mathcal{R})\,(m_-p)^2\,e^{-m_-p} \\
& -\frac{4\,F(m_Y\mathcal{R})}{3}\,(m_Y p)^2\,e^{-m_Y p}\;.
\end{aligned}
$$

Gravity Prob B and LARES



	Measured	Predicted
Geodetic precession (mas)	6602 ± 18	6606
Frame-dragging (mas)	37.2 ± 7.2	39.2



- Gravity Prob B: was a relativity gyroscope experiment funded by NASA which launched on 20 April 2004 and and completed on 8 December 2010. The mission plans were to test two unverified predictions of general relativity: the geodetic effect and frame-dragging. This was to be accomplished by measuring, very precisely, tiny changes in the direction of spin of four gyroscopes contained in an Earth satellite orbiting at 650 *km* altitude, crossing directly over the poles.
- LARES: (acronym for Laser Relativity Satellite) is an Italian Space Agency scientific satellite launched on 13 February 2012.The satellite, completely passive, is made of tungsten alloy and houses 92 cube corner retro reflectors that are used to track the satellite via laser from stations on Earth. LARES's body has a diameter of about 36.4 *cm* and weighs about 400 *Kg*. LARES was inserted in an orbit with 1450 *Km* of perigee, an inclination of 69.5 degrees and reduced eccentricity $\sim 10^{-3}$. The satellite is tracked by the International Laser Ranging Service stations. The main scientific target of the LARES mission is the measurement of the frame-dragging, also known as Lense-Thirring effect, with an accuracy of about 1%.

Experimental constraints

- Geodesic and Lense-Thirring precessions are:

$$\mathbf{\Omega}_{\rm G} = \frac{\nabla(\Phi + 2\Psi)}{2} \times \mathbf{v} = \mathbf{\Omega}_{\rm G}^{\rm (GR)} + \mathbf{\Omega}_{\rm G}^{\rm (STFOG)}, \quad \mathbf{\Omega}_{\rm LT} = \frac{\nabla \times \mathbf{A}}{2} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (STFOG)}$$

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- Gravity Probe B

$$g(\xi,\eta)(m_+\,r^*+1)\,F(m_+R_\oplus)\,e^{-m_+\,r^*}+\frac{8}{3}(m_Y\,r^*+1)\,F(m_YR_\oplus)\,e^{-m_Y\,r^*}+$$
$$+[1/3-g(\xi,\eta)](m_-\,r^*+1)\,F(m_-R_\oplus)\,e^{-m_-\,r^*} \lesssim \frac{3\,\delta|\mathbf{\Omega}_{\rm G}|}{|\mathbf{\Omega}_{\rm G}^{\rm (GR)}|}\simeq 0.008\,,$$

$$(1+m_Yr^*+m_Y{}^2r^{*2})\,e^{-m_Yr^*} \lesssim \frac{\delta|\mathbf{\Omega}_{\rm LT}|}{|\mathbf{\Omega}_{\rm LT}^{\rm (GR)}|}\simeq 0.19 \quad\Rightarrow\quad m_Y \geq 7.3\times10^{-7}m^{-1}$$

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$$+[1/3 - g(\xi,\eta)](m_- r^* + 1)\, F(m_- R_\oplus)\, e^{-m_- r^*} \lesssim \frac{3\,\delta|\mathbf{\Omega}_\mathrm{G}|}{|\mathbf{\Omega}_\mathrm{G}^{(\mathrm{GR})}|} \simeq 0.008\,,$$

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$$(1 + m_Y r^* + {m_Y}^2 r^{*2})\, e^{-m_Y r^*} \lesssim \frac{\delta|\mathbf{\Omega}_\mathrm{LT}|}{|\mathbf{\Omega}_\mathrm{LT}^{(\mathrm{GR})}|} \simeq 0.01 \quad\Rightarrow\quad m_Y \geq 1.2 \times 10^{-6} m^{-1}$$

$r^* = R_\oplus + h$, $R_\oplus$ is the radius of the Earth and h is the altitude of the satellite ($h = 650\ km$ for Gravity Probe B, while $h = 1450 Km$ for LARES).

Noncommutative Spectral Geometry model

- Gravitational part of the asymptotic expression for the bosonic sector of the NCSG action:

$$\mathcal{S}^{\mathrm{L}}_{\mathrm{grav}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2\kappa_0^2} + \alpha_0 C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + \tau_0 R^\star R^\star - \xi_0 R|\mathbf{H}|^2\right]$$

- $C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} = 2R_{\alpha\beta}R^{\alpha\beta} - \frac{2}{3}R^2$.
- $\mathbf{H} = (\sqrt{af_0}/\pi)\phi$; $\alpha_0 = -3f_0/(10\pi^2)$; $\xi_0 = \frac{1}{12}$.
- $m_R \rightarrow \infty$; $m_Y = \sqrt{\frac{5\pi^2\left(k_0^2\mathbf{H}^{(0)}-6\right)}{36f_0k_0^2}}$; $m_\phi = 0$; $\xi = \frac{af_0(\mathbf{H}^{(0)})^2}{12\pi^2}$, $\eta = 0$;
  $g(\xi,\eta) = \frac{af_0(\mathbf{H}^{(0)})^2+12\pi^2}{6|af_0(\mathbf{H}^{(0)})^2-12\pi^2|} + \frac{1}{6}$; $w_\pm = 1 - \frac{af_0(\mathbf{H}^{(0)})^2}{12\pi^2}$, 0.
- Geodesic effect: $\frac{8}{3}(m_Y\,r^* + 1)\,F(m_Y R_\oplus)\,e^{-m_Y\,r^*} \lesssim 0.008 \Rightarrow$, $m_Y \geq 7.1\times 10^{-5}m^{-1}$

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- But... a more stringent constraint has been obtained using torsion balance experiments, $m_Y \geq 10^4 m^{-1}$

Weak field limit of STFOG

- $g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu} \,, \qquad \phi \sim \phi^{(0)} + \phi^{(2)} + \ldots = \phi^{(0)} + \varphi$

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- we can use the harmonic gauge condition $g^{\rho\sigma}\Gamma^{\alpha}{}_{\rho\sigma} = 0$ we set $h_{\mu\sigma}{}^{,\sigma} - 1/2\, h_{,\mu} = 0$,
- The field equations are:

$$(\Box_\eta + {m_Y}^2)\Box_\eta h_{\mu\nu} - \left[\frac{{m_R}^2 - {m_Y}^2}{3{m_R}^2}\,\partial^2_{\mu\nu} + \eta_{\mu\nu}\left(\frac{{m_Y}^2}{2} + \frac{{m_R}^2 + 2{m_Y}^2}{6{m_R}^2}\Box_\eta\right)\right]\Box_\eta h +$$

$$+2\,{m_Y}^2\, f_{R\phi}(0,0,\phi^{(0)})\,(\partial^2_{\mu\nu} - \eta_{\mu\nu}\Box_\eta)\varphi = -2\,{m_Y}^2\,\mathcal{X}\,T_{\mu\nu}$$

$$(\Box_\eta + {m_R}^2)\Box_\eta h + 6{m_R}^2\, f_{R\phi}(0,0,\phi^{(0)})\,\Box_\eta\varphi = 2{m_R}^2\,\mathcal{X}\,T$$

$$(\Box_\eta + {m_\phi}^2)\varphi + \frac{f_{R\phi}(0,0,\phi^{(0)})}{2}\,\Box_\eta h = 0$$

Weak field limit of STFOG

- To solve the system we introduce the auxiliary fields $\gamma_{\mu\nu}$, Γ, Ψ and Ξ such that we can rewrite the metric $h_{\mu\nu}$ as:

$$\begin{aligned} h_{\mu\nu} \quad = \quad & \gamma_{\mu\nu} + \frac{1}{m_Y^2}\left[\frac{{m_R}^2 - {m_Y}^2}{3{m_R}^2}\,\partial^2_{\mu\nu} + \eta_{\mu\nu}\left(\frac{{m_Y}^2}{2} + \frac{{m_R}^2 + 2{m_Y}^2}{6{m_R}^2}\Box_\eta\right)\right]\Gamma + \\ & + f_{R\phi}(0,0,\phi^{(0)})\left[\partial^2_{\mu\nu}\,\Psi + \frac{1}{2}\eta_{\mu\nu}\Xi\right], \end{aligned}$$

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$$\begin{aligned}
(\Box_\eta + {m_Y}^2)\Box_\eta\gamma_{\mu\nu} &= -2\,{m_Y}^2\,\mathcal{X}\,T_{\mu\nu} \\
(\Box_\eta + {m_Y}^2)\Box_\eta\gamma &= -2\,{m_Y}^2\,\mathcal{X}\,T \\
(\Box_\eta + m_+^2)(\Box_\eta + m_-^2)\varphi &= -{m_R}^2 f_{R\phi}(0,0,\phi^{(0)})\,\mathcal{X}\,T \\
(\Box_\eta + {m_R}^2)(\Box_\eta + {m_Y}^2)\Box_\eta\Gamma &= 2\,{m_R}^2\,{m_Y}^2\,\mathcal{X}\,T \\
(\Box_\eta + m_+^2)(\Box_\eta + m_-^2)(\Box_\eta + {m_R}^2)\Box_\eta\Psi &= 2\,{m_R}^4 f_{R\phi}(0,0,\phi^{(0)})\,\mathcal{X}\,T \\
(\Box_\eta + m_+^2)(\Box_\eta + m_-^2)(\Box_\eta + {m_R}^2)\Xi &= 2\,{m_R}^4 f_{R\phi}(0,0,\phi^{(0)})\,\mathcal{X}\,T
\end{aligned}$$

Weak field limit of STFOG

- In far zone limit $|\mathbf{x}| >> \lambda >> |\mathbf{x}'|_{max}$ and for the quadrupole term, we get the solutions:

$$h_{ij}(|\mathbf{x}|,t) = -2\,m_Y\,\Upsilon_{ij}^{m_Y}(|\mathbf{x}|,t) + \eta_{ij}\,\mathrm{H}\Upsilon^{m_Y}(|\mathbf{x}|,t) - \eta_{ij}\,\Pi\,\Upsilon^{m_R}(|\mathbf{x}|,t) + \\ - \sum_{S=\pm}\frac{g_S(\xi,\eta)}{3}\eta_{ij}\,m_S\Upsilon^{m_S}(|\mathbf{x}|,t)$$

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- where

$$\begin{aligned} \Upsilon^m_{ij}(|\mathbf{x}|,t) &= \frac{\mathcal{X}}{24\pi}\int_0^\infty d\tau\,\frac{\mathcal{J}_1(\tau)}{\sqrt{\tau^2+m^2|\mathbf{x}|^2}}\,\ddot{Q}_{ij}(t-\tau_m)\,, \qquad \Upsilon^m(|\mathbf{x}|,t) = \eta^{ij}\Upsilon^m_{ij}(|\mathbf{x}|,t) \\ \mathrm{H} &= g_Y\,m_Y = \frac{3\,m_R^2 m_Y}{(m_R^2-m_Y^2)}\,, \qquad \Pi = g_R\,m_R = \frac{(2\,m_R^2+m_Y^2)\,m_R}{(m_R^2-m_Y^2)} \\ g_\pm(\xi,\eta) &= \frac{\xi}{[w_\mp(\xi,\eta)-w_\pm(\xi,\eta)]\,[1-w_\pm(\xi,\eta)]}\,, \quad \tau_m = \frac{\sqrt{\tau^2+m^2|\mathbf{x}|^2}}{m} \end{aligned}$$

Weak field limit of STFOG

- The rate of energy loss from a binary system, in the far-field limit, is given by:

$$-\frac{d\mathcal{E}}{dt} \approx \frac{2\,\pi\,|\mathbf{x}|^2}{5\,\mathcal{X}}\,\dot{h}_{ij}\,\dot{h}^{ij} = \dot{\mathcal{E}}_{GR} + \dot{\mathcal{E}}_{STFOG}$$

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- Let's consider the following time dependence of the quadrupole moment and its spatial trace:

$$Q_{ij}(t) = \mathcal{Q}_{ij}\cos(\omega_{(ij)}\,t + \vartheta_{(ij)}) + \mathcal{Q}^0_{ij}\,, \quad Q(t) = \mathcal{Q}\cos(\nu\,t + \vartheta) + \mathcal{Q}^0$$

$$\dot{\mathcal{E}}_{GR} = \frac{\mathcal{X}}{720\,\pi}\left(\omega^6_{(ij)}\,\mathcal{Q}_{ij}\,\mathcal{Q}^{ij} - \frac{\nu^6\,\mathcal{Q}^2}{4}\right)$$

$$\dot{\mathcal{E}}_{STFOG} = \frac{\mathcal{X}}{720\,\pi}\left[\omega^6_{(ij)}\,\mathcal{Q}_{ij}\,\mathcal{Q}^{ij} + \frac{\nu^6\,\mathcal{Q}^2}{4}\,\zeta_{YY}\right]\Lambda\left(\omega^c_{YY}|\mathbf{x}|;\,\frac{\omega_{(ij)}}{\omega^c_{YY}}\right)$$

$$+\frac{\nu^6\,\mathcal{Q}^2\,\mathcal{X}}{2880\,\pi}\left[\zeta_{RR}\,\Lambda\left(\omega^c_{RR}|\mathbf{x}|;\,\frac{\nu}{\omega^c_{RR}}\right) + (\zeta_{--} + \zeta_{R-})\,\Lambda\left(\omega^c_{--}|\mathbf{x}|;\,\frac{\nu}{\omega^c_{--}}\right)\right.$$

$$\left. + \zeta_{YR}\,\Lambda\left(\omega^c_{YR}|\mathbf{x}|;\,\frac{\nu}{\omega^c_{YR}}\right) + \zeta_{Y-}\,\Lambda\left(\omega^c_{Y-}|\mathbf{x}|;\,\frac{\nu}{\omega^c_{Y-}}\right)\right]$$

$$\Lambda(x;y) = \frac{\mathcal{J}_1(x-y)}{x\,(1-y)}\,,\ \zeta_{YY} = g_Y\,(3\,g_Y - 4)\,,\quad \zeta_{RR} = 3\,g_R^2\,,\quad \zeta_{--} = \frac{g_-^2}{3}$$

$$\zeta_{YR} = (2 - 3\,g_Y)\,g_R\,,\ \zeta_{Y-} = \frac{1}{3}\,(2 - 3\,g_Y)\,g_-\,,\ \zeta_{R-} = g_R\,g_-$$

$$\omega^c_{YY} = m_Y\,,\ \omega^c_{RR} = m_R\,,\ \omega^c_{--} = m_R\sqrt{w_-}\,,\ \omega^c_{YR} = \sqrt{m_Y\,m_R}\,,\ \omega^c_{Y-} = \sqrt{m_Y\,m_R}\sqrt{w_-}$$

Weak field limit of STFOG

- As example let's consider a pair of masses m_1 and m_2 ($\mu = \frac{m_1\,m_2}{m_1+m_2}$) in an elliptic binary system. For such system the only nonzero components of the quadrupole are:

$$\mathcal{Q}_{xx} = \frac{3}{2}\,\mu\,a^2\,,\quad \mathcal{Q}_{yy} = -\frac{3}{2}\,\mu\,b^2\,,\quad \mathcal{Q}_{xy} = \frac{3}{2}\,\mu\,a\,b\,,\quad \mathcal{Q} = -\frac{3}{2}\,\mu\,a^2\,e^2$$

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- The variation of period is connected to energy loss:

$$\frac{d\mathcal{P}}{dt} = \frac{\mathcal{P}^3}{4\pi I_{PSR}}\,\frac{d\mathcal{E}}{dt}$$

where I_{PSR} is the pulsars moment of inertia, normally assumed to be 10^{45}g cm^2

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- The variation of period is connected to energy loss:

$$\frac{d\mathcal{P}}{dt} = \frac{\mathcal{P}^3}{4\pi I_{PSR}}\frac{d\mathcal{E}}{dt}$$

where I_{PSR} is the pulsars moment of inertia, normally assumed to be 10^{45}g cm^2

$$\begin{aligned}\frac{\dot{\mathcal{P}}_{STFOG}}{\dot{\mathcal{P}}_{GR}} = \frac{4\left(a^2+b^2\right)^2+a^4\,e^2\,\zeta_{YY}}{4\left(a^2+b^2\right)^2-a^4\,e^2}\,\Lambda\left(m_Y\,|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{YY}}\right) + \frac{a^4\,e^2}{4\left(a^2+b^2\right)^2-a^4\,e^2}\Bigg[\zeta_{RR}\,\Lambda\left(m_R\,|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{RR}}\right) +\\ +(\zeta_{--}+\zeta_{R-})\,\Lambda\left(m_-\,|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{--}}\right) + \zeta_{YR}\,\Lambda\left(\sqrt{m_Y\,m_R}|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{YR}}\right) +\\ +\zeta_{Y-}\,\Lambda\left(\sqrt{m_Y\,m_-}|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{Y-}}\right)\Bigg]\end{aligned}$$

Weak field limit of STFOG: Constraint

- PSR J0348+0432[1] is a neutron star in a binary system with a white dwarf has an almost circular orbit, a semi major axis $a = 8.3 \times 10^8$ m and a short orbital period $\mathcal{P} = 2.46$ hours orbit. The observers obtained the constraint $\dot{\mathcal{P}}_{obs}/\dot{\mathcal{P}}_{GR} = 1.05 \pm 0.18$. We get the experimental condition:

$$-0.13 \leq \frac{\dot{\mathcal{P}}_{STFOG}}{\dot{\mathcal{P}}_{GR}} \leq 0.23 \quad \Rightarrow \quad -0.13 \leq \Lambda\left(m_Y\,|\mathbf{x}|;\ \frac{2\,\Omega}{\omega^c_{YY}}\right) \leq 0.23$$

$$m_Y > 5 \times 10^{-11}\, m^{-1}$$

[1] Antoniadis J. et al, A Massive Pulsar in a Compact Relativistic Binary. *Science* 2013, **340**, 1233232 (10).

[2] Hulse R.A., Taylor J.H., Discovery of a pulsar in a binary system. *Astrophysical Journal* 1975, **195**, L51 (3)

[3] Weisberg J.M., Nice D.J., Taylor J.H., Timing Measurements of the Relativistic Binary Pulsar PSR B1913+16. *Astrophysical Journal* 2010, **722**, 1030 (5).

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- PSR B1913+16[2] [3] was discovered by Hulse and Taylor in 1974, the the experimental eccentricity $e = 0.6$, semi major axis $a = 1.95 \times 10^9$ m, the orbital period $\mathcal{P} = 7.7$ hours orbit and $\dot{\mathcal{P}}_{obs}/\dot{\mathcal{P}}_{GR} = 0.997 \pm 0.002$:

$$-0.005 \leq \frac{\dot{\mathcal{P}}_{STFOG}}{\dot{\mathcal{P}}_{GR}} \leq -0.001$$

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Weak field limit of STFOG: Constraint

Case	STFOG Theory	Mass definition	Constraint
A	$f(R)$ $R - \frac{R^2}{R_0}$ [4]	$m_R{}^2 = -\frac{f_R(0)}{3f_{RR}(0)}$ $m_Y{}^2 \to \infty, \quad m_\phi{}^2 = 0$ $\xi = 0, \quad \eta = 0$ $m_+ = m_R, \quad m_- = 0$	$m_R \gtrsim 3 \times 10^{-9}\text{m}^{-1}$ $m_R = \sqrt{-\frac{a_1}{6\,a_2}}$ $R_0 \gtrsim 5 \times 10^{-19}\text{m}^{-2}$
B	$f(R,\ R_{\alpha\beta}R^{\alpha\beta})$ $R + a_0 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} =$ $R + 2a_0[R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2]$	$m_R{}^2 = -\frac{f_R(0)}{3f_{RR}(0,0)+2f_Y(0,0)}$ $m_Y{}^2 = \frac{f_R(0)}{f_Y(0,0)}, \quad m_\phi{}^2 = 0$ $\xi = 0, \quad \eta = 0$ $m_+ = m_R, \quad m_- = 0$	$m_R \gtrsim 1 \times 10^{-9}\text{m}^{-1}$ $m_Y \gtrsim 5 \times 10^{-11}\text{m}^{-1}$ $a_0 \lesssim 1.2 \times 10^{20}\ \text{m}^2$

[4] Starobinsky A. A., Phys. Lett. B **91**, 99 (1980)

Weak field limit of STFOG: Constraint

Case	STFOG Theory	Mass definition
C	$f(R,\ R_{\alpha\beta}R^{\alpha\beta},\phi)+\omega(\phi)\phi_{;\alpha}\phi^{;\alpha}$	$m_R{}^2 = -\frac{f_R(0,0,\phi^{(0)})}{3f_{RR}(0,0,\phi^{(0)})+2f_Y(0,0,\phi^{(0)})}$ $m_Y{}^2 = \frac{f_R(0,0,\phi^{(0)})}{f_Y(0,0,\phi^{(0)})},\quad m_\phi{}^2 = -f_{\phi\phi}(0,0,\phi^{(0)})$ $\xi = 3f_{R\phi}(0,0,\phi^{(0)})^2,$

- Noncommutative Spectral Geometry

$$m_\phi \gtrsim 1.3\times 10^{-11}\ \mathrm{m}^{-1} \qquad \alpha_0 \lesssim 1.2\times 10^{20}\ \mathrm{m}^2$$

- In a curved space-time the massless scalar field ϕ obeys the following field equation[5]:

$$\left[\Box + \varepsilon\,\mathcal{R}(\mathbf{x})\right]\phi \;=\; \left[\frac{1}{\sqrt{-g}}\,\partial_\alpha\,\sqrt{-g}\,g^{\alpha\beta}\,\partial_\beta + \varepsilon\,\mathcal{R}(\mathbf{x})\right]\phi \;=\; 0$$

[5] Sorge F., Casimir effect in a weak gravitational field, Class. Quantum Grav. 22 (2005) 5109-5119

Casimir Effect

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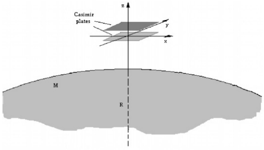

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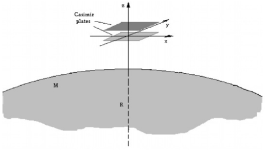

$$\begin{aligned} g_{00}(\mathbf{x}) &\simeq 1 + 2\,\Phi_0(R) + 2\,\Lambda(R)\,z\,, \\ g_{ij}(\mathbf{x}) &\simeq -1 + 2\,\Psi_0(R) + 2\,\Sigma(R)\,z\,, \\ \mathcal{R}(\mathbf{x}) &\simeq \mathcal{R}_1(R) + \mathcal{R}_2(R)\,z\,, \end{aligned}$$

$$\ddot{\phi} - \left[1 + 4\Phi + 4\gamma z\right]\,\triangle\,\phi + \varepsilon\left[\mathcal{R}_1 + \mathcal{R}_2 z\right]\phi \;=\; 0\,,$$

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Casimir Effect

- The mean vacuum energy density between the plates:

$$\langle e \rangle = A\left[-\frac{\pi^2}{1440\,L_p^4}B + \varepsilon\frac{\mathcal{R}_1}{96\,L_p^2}C\right]$$

where

$$\begin{aligned} A &= 1+\Phi+(\gamma+\Sigma)L \\ B &= 1+12\Phi-8\Psi+6\gamma L \\ C &= 1-8\Phi_0-6\Psi_0-2\gamma L+2\Sigma L \end{aligned}$$

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- The typical experimental the error whit $L = 100nm$ is approximately equal to only 0.03% [6]

$$\frac{G\,M\,g(\xi,\eta)\,F(m_+R_\oplus)e^{-m_+\,R_\oplus}\left[R_\oplus+(1+m_+\,R_\oplus)L\right]}{R^2}+$$
$$+\frac{G\,M\,\left(g(\xi,\eta)-\frac{1}{3}\right)\,F(m_-\,R_\oplus)e^{-m_-\,R_\oplus}\left[R_\oplus+(1+m_-\,R_\oplus)L\right]}{R^2}+$$
$$+\frac{4\,G\,M\,F(m_Y R_\oplus)e^{-m_Y\,R_\oplus}\left[R_\oplus+2(1+m_Y\,R_\oplus)L\right]}{3R^2}\lesssim 0.003$$

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Thanks for your attention