Semi-analytical approach versus simulations for e-cloud effects in the LHC magnetic dipole

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Outline

1. Electron Cloud in LHC
2. Map Formalism
3. The linear coefficient
4. PyEcloud Outputs
5. Theoretical model for e-cloud distribution
6. Calculation of saturation density and quadratic coefficient
7. Conclusions
The Multipacting Effect

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Quantities</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam pipe radius (circular case)</td>
<td>$R_P$</td>
<td>m</td>
<td>0.020</td>
</tr>
<tr>
<td>Beam pipe radii (elliptic case)</td>
<td>$R_{p1}, R_{p2}$</td>
<td>m</td>
<td>0.022, 0.017</td>
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<tr>
<td>Beam size</td>
<td>$\sigma_r$</td>
<td>m</td>
<td>0.002</td>
</tr>
<tr>
<td>Bunch spacing</td>
<td>$s_b$</td>
<td>m</td>
<td>7.480</td>
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<tr>
<td>Bunch length</td>
<td>$\sigma_z$</td>
<td>m</td>
<td>0.023</td>
</tr>
<tr>
<td>Particles per bunch</td>
<td>$N_b$</td>
<td>$10^{10}$</td>
<td>8 ÷ 12</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$B$</td>
<td>T</td>
<td>8</td>
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</tbody>
</table>
Maps Formalism in presence of magnetic field

The bunch to bunch evolution of the e-cloud density can be represented by a cubic map:

\[ \lambda_{m+1} = a\lambda_m + b\lambda_m^2 + c\lambda_m^3 \]

Lines correspond to cubic fit:

\[ \lambda_{m+1} = a\lambda_m + b\lambda_m^2 + c\lambda_m^3 \]

It is necessary to determine an analytical form for the three coefficients \( a, b \) and \( c \)

\( a \) is related to the build up of the cloud while \( b \) is related to the saturation.

During the saturation, neglecting the term \( c \):

\[ \lambda_{m+1} = \lambda_m = \lambda_{sat} \rightarrow b = \frac{1 - a}{\lambda_{sat}} \]
Analytical determination of linear coefficient

The analytical form of $a$ is provided by U. Iriso and S. Peggs (2005)

$$a = \delta_{\text{ref}}(E_g)^k + \delta_{ts}(E_g)\delta_{\text{tot}}(E_0)\xi \times \frac{\delta_{\text{tot}}(E_0)^k\xi - \delta_{\text{ref}}(E_g)^k}{\delta_{\text{tot}}(E_0)^\xi - \delta_{\text{ref}}(E_g)}$$

$$\xi = \sqrt{\frac{E_0}{E_g}}$$

$E_0 \rightarrow$ secondary electron energy

$E_g \rightarrow$ energy gained by the electrons after the passage of the bunch

$$\delta_{\text{max}}(\theta) = \delta_{\text{max}} e^{\frac{1 - \cos \theta}{2}}$$

$$\delta_{ts}(E) = \delta_{\text{max}}\frac{sE/E_{\text{max}}}{s-1+(E/E_{\text{max}})^s}$$

$$\delta_{\text{ref}}(E) = R_0\left(\frac{\sqrt{E} - \sqrt{E+E_0}}{\sqrt{E} + \sqrt{E+E_0}}\right)^2$$

$$\delta_{\text{tot}}(E) = \delta_{ts}(E) + \delta_{\text{ref}}(E)$$

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{\text{max}}$</td>
<td>/</td>
<td>1.5 ÷ 1.7</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>eV</td>
<td>332</td>
</tr>
<tr>
<td>$s$</td>
<td>/</td>
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<tr>
<td>$E_0$</td>
<td>eV</td>
<td>150</td>
</tr>
<tr>
<td>$R_0$</td>
<td>/</td>
<td>0.7</td>
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</table>
Average Cloud Distribution

Average Distribution during saturation

3D Average Distribution during saturation
Theoretical model for e-cloud density distribution

The electrostatic potential \( \nu(r, \phi) \) generated by a negative uniform charged wirelike distribution \((-e \lambda_e)\) satisfying the boundary condition \( \nu(1, \phi) = 0 \) on the chamber wall is given by:

\[
\nu(r, \phi) = -\frac{e \lambda_e}{4\pi \epsilon_0} \ln \frac{r^2 r'^2 - 2rr' \cos(\phi' - \phi) + 1}{r^2 - 2rr' \cos(\phi' - \phi) + r'^2} = -\frac{e \lambda_e}{2\pi \epsilon_0} f(r, \phi, r', \phi')
\]

By the observing of previous outputs we have formulated the following model of distribution:

\[
g(x, y) = \frac{e}{\int_{S'/4} dS'} \frac{(x-x_p)^2}{2\sigma_x^2} e^{-\frac{(x-x_p)^2}{2\sigma_x^2}} \frac{(y-y_p)^2}{2\sigma_y^2} e^{-\frac{(y-y_p)^2}{2\sigma_y^2}}
\]
Energy barrier and saturation condition

The total electrostatic potential considering also the contribution of the bunch is given by:

\[ V(r, \phi) = -\frac{e\lambda}{2\pi\epsilon_0} \int_{s'} dS' g'(r', \phi') v(r, \phi) + \frac{e\bar{\lambda}_b}{2\pi\epsilon_0} h_b(r) \]

where \( h_b(r) = \left( \frac{1}{2} - \frac{r^2}{2\bar{\sigma}^2 r} - \ln \bar{\sigma} r \right) \Theta(\bar{\sigma} r - r) - (\ln r) \Theta(r - \bar{\sigma} r) \), \( \Theta \) is the Heaviside function and \( \bar{\sigma} = \frac{\sigma_r}{R_p} \) with \( \sigma_r \) the radius of bunch and \( R_p \) the radius of pipe.

The energy barrier is given by \(-eV(r, \phi)\), then we find:

\[ \mathcal{E}(r, \phi) = 2r_e m_e c^2 \left\{ \lambda_e h(r, \phi) - \bar{\lambda}_b h_b(r) \right\} \]

The saturation condition can be obtained by imposing that

\[ \mathcal{E}_{\text{max}} = \mathcal{E}(\sqrt{x_p^2 + y_p^2}, \arctan y_p / x_p) \gtrsim \mathcal{E}_0 \]

where \((x_p, y_p)\) is the point of the transverse plane to the bunch corresponding to the maximum of energy barrier.
Results

We obtain the density of saturation as follows:

\[
\lambda_{e}^{sat} = \frac{4 \left[ \frac{\varepsilon_0}{m_e c^2} \frac{N_b}{s_b + \sigma_z} \ln \sqrt{x_p^2 + y_p^2} \right]}{\int_{S'/4} dS' g(x', y') f\left( \sqrt{x_p^2 + y_p^2}, \arctan y_p / x_p, x', y' \right)}
\]
Conclusions

- Simple model that doesn't require high computational costs like simulations;
- A theoretical framework for the space charge is missing. Only by the study of the cloud we can obtain the values for model free parameters;
- The model is too simple, since the saturation density doesn't depend on the SEY, while the values obtained by simulations are SEY depending.
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7. G. Iadarola, G. Rumolo - PyECLoud and build-up simulation at CERN - Proceedings of ECloud12