\[ f(x) = \begin{cases} -e^{-x^2} & -1 \leq x < 0 \\ e^{-x^2} & 0 \leq x < 1 \end{cases} \]

\[ T = 2, \quad \omega = \frac{2\pi}{T} = \pi \]

\[ f = \text{odd function}, \quad c_n = 0 \]

\[ b_k = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin k\omega x \, dx = \frac{4}{\pi} \int_{0}^{T/2} e^{-x^2} \sin k\pi x \, dx = \frac{8}{\pi} \int_{0}^{T/2} e^{-x^2} \sin (k\pi x) \, dx \]

Noting that

\[ \int_{0}^{T/2} e^{-x^2} \sin (k\pi x) \, dx = -\frac{e^{-x^2} \sin (k\pi x)}{k\pi} + \frac{1}{k\pi} \int_{0}^{T/2} e^{-x^2} \cos (k\pi x) \, dx \]

\[ = -\frac{e^{-x^2} \sin (k\pi x)}{k\pi} + \frac{1}{k\pi} \left( \frac{e^{-x^2} \cos (k\pi x)}{k\pi} - \frac{1}{k\pi} \int_{0}^{T/2} e^{-x^2} \cos (k\pi x) \, dx \right) \]

\[ \Rightarrow \int_{0}^{T/2} e^{-x^2} \sin (k\pi x) \, dx = \left( d + \frac{1}{k^2\pi^2} \right) \left[ \frac{e^{-x^2} \sin (k\pi x) - \frac{1}{k\pi} e^{-x^2} \cos (k\pi x)}{k^2\pi^2} \right] \]

\[ b_k = \frac{2}{\pi} \frac{u\pi}{1 + u^2\pi^2} \left( 1 + e^{-(u+d)^2} \right) \]
\[ f(x) = \sum_{n=1}^{\infty} \frac{2 \pi n}{1 + n^2 \pi^2} \left[ 1 + C(-1)^n \right] \sin n \pi x \]

\[ \nabla \cdot \vec{A} = \frac{1}{\rho^2 \frac{\partial}{\partial \varphi}} (\rho^2 A_{\varphi}) + \frac{1}{\rho \sin \varphi} \frac{\partial}{\partial \theta} (\rho \sin \varphi A_{\theta}) + \frac{1}{\rho \sin \varphi} \frac{\partial}{\partial \varphi} \frac{\partial}{\partial \varphi} \]

\[ \rho^2 A_{\varphi} = 3 \rho^4 \quad \text{sint} A_{\theta} = 2 \rho \cos \varphi \sin \theta \]

\[ \frac{\partial}{\partial \varphi} \]

\[ \nabla \cdot \vec{A} = \frac{1}{\rho^2} \frac{\partial}{\partial \varphi} (3 \rho^4) + \frac{1}{\rho \sin \varphi} \frac{\partial}{\partial \theta} (2 \rho \cos \varphi \sin \theta) + \frac{1}{\rho \sin \varphi} \frac{\partial}{\partial \varphi} \frac{\partial}{\partial \varphi} \]

\[ = \frac{1}{\rho^2} (12 \rho^2) + \frac{1}{\rho \sin \varphi} \left( 2 \rho \sin^2 \varphi + 2 \rho \cos^2 \varphi \right) + \frac{c_8 \varphi \sin \varphi}{\sin \varphi} \]

\[ = 12 \rho - 2 \sin \theta + 2 \frac{\varphi \cos \varphi}{\sin \theta} + \frac{c_8 \varphi \sin \varphi}{\sin \theta} \]

\[ \vec{r} = (3\sqrt{2}, -3\sqrt{2}, 4) \]

\[ \rho = \sqrt{18 + 18 + 16} = \sqrt{52} = 2\sqrt{13} \]

\[ \theta = \arccos \left( \frac{4}{2\sqrt{13}} \right) = \arccos \frac{2\sqrt{13}}{13} \]

\[ \varphi = \arcsin \left( -\frac{3\sqrt{2}}{6} \right) = -\arcsin \frac{\sqrt{2}}{2} = -\frac{\pi}{4} \]

\[ \nabla \cdot \vec{A} = 34\sqrt{2} - 2\sqrt{1 - \left( \frac{2\sqrt{13}}{13} \right)^2} + \frac{\left( \frac{2\sqrt{13}}{13} \right)^2}{1 - \left( \frac{2\sqrt{13}}{13} \right)^2} + \frac{-\left( \frac{2\sqrt{13}}{13} \right)^2}{1 - \left( \frac{2\sqrt{13}}{13} \right)^2} \]
\[= 24 \sqrt{3} - 2 \sqrt{169 - 52} \cdot \frac{52}{169} + 2 \frac{52}{\sqrt{169 - 52} \cdot 169} - 1/2 \sqrt{169 - 52} \cdot 169\]

\[= 24 \sqrt{3} - \frac{2}{13} \sqrt{117} + 2 \frac{52}{13 \sqrt{117}} - \frac{1}{2} \frac{13}{\sqrt{117}} = \ldots = 6 \sqrt{3} \]

3) \[\Sigma_\theta(z) = (R \cos \theta, R \sin \theta, z)\]

\[\Sigma_{\theta} = (-R \sin \theta, R \cos \theta, 0)\]

\[\Sigma_z = (0, 0, 1)\]

\[\Sigma_\theta \times \Sigma_z = (-R \sin \theta, R \cos \theta, 0) \times (0, 0, 1) = \]

\[= (R \cos \theta, -R \sin \theta, 0)\]

\[\hat{n}(\theta, z) = \frac{\Sigma_\theta \times \Sigma_z}{|\Sigma_\theta \times \Sigma_z|}\]

\[\hat{n}_\theta = (0, 0, 1)\] \quad \text{vector perpendicular to plane}

\[\hat{n}_z = (0, 0, -1)\]

\[F(z) = \left( -\frac{R \sin \theta}{R^2 + 3}, \frac{R \cos \theta}{R^2 + 3}, 1 \right) = \left( -\frac{\sin \theta}{R^2 + 3}, \frac{\cos \theta}{R^2 + 3}, 1 \right)\]

\[F(\theta) = \int F_0 \hat{n}(\theta, z) \text{ Rôle d'\theta} + \int F_0 \hat{n}_\theta \text{ pol \theta} d\theta + \]

\[\mathcal{F}(F) = \int F_0 \hat{n}(\theta, z) \text{ Rôle d'\theta} + \int F_0 \hat{n}_\theta \text{ pol \theta} d\theta + \]

\[+ \int F_0 \hat{n}_z \text{ pol \theta} = \int \left( \frac{\sin \theta \cdot R \cos \theta + \sin \theta \cdot R \sin \theta}{R^2} \right) \text{ Rôle d'\theta} + \]

\[+ \int \left( \frac{\sin \theta \cdot R \cos \theta - \sin \theta \cdot R \sin \theta}{R^2} \right) \text{ pol \theta}\]
\[ + \int_{\Sigma_0} \text{pol} \, d\theta + \int_{\Sigma_1} \text{pol} \, d\theta = \]

\[= \frac{1}{R} \int_{0}^{2\pi} \left( -\sin \phi \, e^{i\theta} + c + e^{i\theta} \sin \phi \right) \, d\theta = 0 \]

Riportando precedentemente il flusso in un campo "unifor- 
me no" nel piano \(xy\), ad un campo "uniforme" lungo \(z\).

\[f(t) = \ln t = \ln(x + i\cdot y) = \ln(\rho \cdot e^{i\theta}) = \]

\[= \ln \rho + i\cdot \theta; \]

\[\frac{\partial f}{\partial x} = \frac{1}{\rho} \frac{\partial f}{\partial \rho} \quad \Rightarrow \quad \frac{1}{x + i\cdot y} = \frac{1}{\rho} \cdot \frac{i}{\rho} \quad \Rightarrow \quad \frac{1}{x + i\cdot y} \leq \frac{1}{\rho} \leq \frac{1}{\rho} \quad \Rightarrow \quad \frac{1}{i\cdot \rho} \leq \]

\[\int_{0}^{\infty} \left. \frac{\sin \phi}{x} \right|_{x} \, dx \rightarrow \int_{0}^{\infty} \frac{\sin \phi}{x} \, dx \rightarrow \int_{0}^{\infty} \frac{\sin \phi}{x} \, dx \]
\[+ \int_{-\infty}^{\infty} e^{i\omega t} dt = 0\]

Nel \( R \rightarrow 0 \int_{-\infty}^{\infty} e^{i\omega t} dt \rightarrow 0 \) \( (\text{Teorema principale}) \)

Nel \( \varepsilon \rightarrow 0 \int_{-\varepsilon}^{\varepsilon} e^{i\omega t} dt \neq 0 \) \( (\text{non verificare teorema}) \)

\[\int_{-\infty}^{\infty} \frac{e^{ix}}{x} dx + \int_{-\infty}^{\infty} \frac{e^{-ix}}{x} dx = -\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{\omega t} dt\]

\[\varepsilon \varepsilon \Rightarrow \varepsilon e^{i\theta} \Rightarrow \frac{1}{\varepsilon} \int_{0}^{\varepsilon} \frac{1}{t} dt = i\varepsilon e^{i\theta} \int_{0}^{\varepsilon} \frac{1}{t} dt = i\varepsilon \log \varepsilon\]

\[\frac{-i\omega t}{\omega t} \rightarrow \int_{0}^{\pi} \frac{1}{t} dt = \int_{0}^{\pi} \frac{1}{\tan^{2} \theta} d\theta = \int_{0}^{\pi} \frac{1}{\sin^{2} \theta} d\theta = -\pi\]

\[i \int_{0}^{\pi} (1 + i\varepsilon e^{i\theta}) d\theta \rightarrow i \int_{0}^{\pi} d\theta = -\pi\]

\[-\int_{-\varepsilon}^{\varepsilon} \frac{e^{ix}}{x} dx + \int_{-\varepsilon}^{\varepsilon} \frac{e^{-ix}}{x} dx = \int_{-\varepsilon}^{\varepsilon} \frac{e^{-i\omega x}}{-\omega x} dx + \int_{-\varepsilon}^{\varepsilon} \frac{e^{i\omega x}}{\omega x} dx \]

\[-\int_{\varepsilon}^{R} \frac{e^{-i\omega x}}{x} dx + \int_{\varepsilon}^{R} \frac{e^{i\omega x}}{x} dx = \int_{\varepsilon}^{R} \frac{e^{-i\omega x}}{x} dx + \int_{\varepsilon}^{R} \frac{e^{i\omega x}}{x} dx = \int_{\varepsilon}^{R} \frac{2i\sin \omega x}{x} dx\]
\[ \int_{0}^{\infty} \frac{\sin mx}{x} \, dx = \pi \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \]

\[ R \to \infty \quad \text{as} \quad R \to \infty \]

\[ \int_{0}^{\infty} \frac{\sin mx}{x} \, dx = \pi - \ln \left( \frac{m}{\pi} \right) \]

\[ \int_{0}^{\infty} \frac{\sin mx}{x} \, dx = \frac{\pi}{2} \]

\[ \int_{0}^{\infty} \frac{\sin mx}{x} \, dx = \pi \]

\[ y'' + \gamma = \frac{2}{5} \]

\[ y(0) = 0 \quad \Rightarrow \quad y' + \frac{2}{5} y = -\frac{2}{5} \]

\[ y = \frac{2}{5} \left( \frac{1}{s^3} + \frac{1}{s^4} \right) \quad \Rightarrow \quad y = \frac{2}{5} t^{-\frac{3}{5}} \]

\[ y(t) = \frac{2}{5} \int_{0}^{t} e^{-\frac{3}{5} s} \left( 1 - e^{-\frac{3}{5} s} \right) \, ds \]

\[ y(t) = \frac{2}{3} t - \frac{2}{5} t^{rac{3}{5}} \]

\[ f(x) = \begin{cases} \sin x - \cos 2x & \text{if } x > 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x < 0 \end{cases} \]

\[ \hat{f}(u) = \int_{-\infty}^{\infty} \sin x e^{-iux} \, dx = \int_{-\infty}^{0} \sin x e^{-iux} \, dx + \int_{0}^{\infty} \sin x e^{-iux} \, dx \]

\[ = \int_{0}^{\infty} \left( e^{-iux} - e^{iux} \right) \frac{e^{-ix}}{2i} \, dx = \frac{1}{2i} \int_{0}^{\infty} \sin x e^{-i(x+ux)} \, dx + \frac{1}{2i} \int_{0}^{\infty} \cos x e^{-i(x+ux)} \, dx = \frac{1}{2i} \int_{0}^{\infty} i(x+ux) e^{-i(x+ux)} \, dx + \frac{1}{2i} \int_{0}^{\infty} e^{-i(x+ux)} \, dx \]
\[ -\frac{1}{2} \int_{-\infty}^{+\infty} e^{-i(2-k)x} \, dx -\frac{1}{2} \int_{-\infty}^{+\infty} e^{-i(2+k)x} \, dx = \]

\[ = \frac{1}{2i} \left( \delta(1-u) - \delta(1+u) \right) - \frac{1}{2} \left( \delta(2-u) + \delta(2+u) \right). \]

\[ \delta \int_{-1}^{1} \left( y_1^2 + xy^2 - 2cty \right) \, dx = 0 \]

\[ \frac{d}{dx} \frac{\partial L}{\partial y} - \frac{\partial L}{\partial y} = 0 \quad 2y'' - 2xy' = 0 \]

\[ y'' - xy = 0 \quad x < 0 \quad y = A \cos \sqrt{x} + B \sin \sqrt{x} \]

\[ y'' - xy = 0 \quad x > 0 \quad y = A e^{-\sqrt{x}} + B e^{\sqrt{x}} \]

\[ x > 0 \quad y = A e^{-\sqrt{x}} + B e^{\sqrt{x}} \]